



The Tradeoff Theory as a Method of Choosing Capital Structure: Effects of Debt and Tax on the Firm Value

Valery Shemetov

Abstract: We analyze the trade-off theory and its underpinnings. All those papers commit the same error: While analyzing the influence of debt and tax on the firm value, they do not consider debt redemption. This approach turns the firm into a generator of arbitrage profits, making void all results achieved in the papers. Using the Extended Merton model, which takes account of debt, its redemption, tax, and the tax shield, we compute the asset distributions for an all-equity firm and the identical levered firm. Contrary to the general belief that a medium-size debt is beneficial for the firm, characteristics of the levered firm are inferior to the same characteristics of the unlevered firm. The anticipated gains from tax deductions are the phantoms created by errors in the papers. Our analysis of debt and tax on the firm value can be helpful for managers as a method of computing the consequences of capital structure decisions.

Keywords: Trade-off theory, debt, corporate income tax, tax deductibles, levered/unlevered firm, Extended Merton model (EMM)

INTRODUCTION

Modern theories of capital structure start with the seminal Modigliani-Miller Propositions (hereafter MMPs). The MMP1 (Modigliani & Miller 1958) states that the expected firm value does not depend on the capital structure in the perfect market. The MMP3 (Modigliani & Miller 1963) considers an all-equity (unlevered) firm and the levered firm identical to the first one in all respects but the capital structure. The MMP3 argues that in the perfect market with corporate income taxes and tax deductibles, the after-tax expected value of the levered firm equals the after-tax value of the unlevered firm plus tax deductions. Kraus and Litzenberger (1973) put forward the trade-off hypothesis that in the perfect market with corporate taxes and bankruptcy costs, there is an optimal debt level maximizing the firm value. Brennan and Schwartz (1978) and Bradley, Jarrell, and Kim (1984) start development of the static trade-off theory, but by general opinion, the main contribution to this theory is made by Leland (1994) and Leland and Toft (1996).

Leland considers the firm and a security representing a claim on the firm. The firm continuously pays for a coupon per instant of time until it comes to bankruptcy. He derives a closed-form solution for the firm's debt, equity, and value. His firm value is equal to the asset value, plus the tax deductions of coupon payments, less the value of bankruptcy costs. This relation makes the basis of the static trade-off theory, which Leland applies to determine the optimal capital structure in the exogenous default. He has also shown that if the firm's managers can choose the time of default, they can do it, maximizing the firm's equity at this time (the endogenous default). However, his optimal debt level is extremely high (75-90%), while a typical firm has a debt ratio in the 20-40% range (Beattie et al. 2006). This difference signals that the author has missed important phenomena in his analysis of debt. Besides, Leland's problem setting has a serious restriction: the debt maturity is

infinite. Leland and Toft (1996) revisit the problem, considering the debt of fixed maturity. However, their model still shows too high levels of the optimal debt. Myers (1984), analyzing the static trade-off theory, remarks: 'People feel comfortable with the static trade-off story because it sounds plausible and rationalizes moderate borrowing. ... We have to ask whether it explains firms' financing behavior. If it does, fine. If it does not, then we need a better theory before offering advice to managers. (p. 588)'

Failing to achieve realistic levels of debt with static trade-off models, researchers turn to the dynamic models, anticipating that dynamic debt adjustment can reconcile the theory with practice. One can find a wide panorama of those attempts in a comprehensive review of dynamic models by Strebulaev and Whited (2012). The models in this review are based on the GBM equation (Merton 1974). The authors assert that the GBM equation is good for describing the 'stock price in the Black-Scholes model; firm value, firm cash flows, or prices of firm output and input in other corporate finance settings' (Strebulaev & Whited 2012, pp. 4-5). Sundaresan (2013, p. 21), echoing them, states that 'since its publication, the seminal structural model of default by Merton (1974) has become the workhorse for gaining insights about how firms choose their capital structure.' The dynamic models provide for lesser debt ratios than the static ones, but still higher than those observed in practice.

Our objective is to show that both static and dynamic trade-off theories are wrong, and their principal error is in using the GBM equation for description of the firm value dynamic. The GBM equation does not include any payments or investments in the firm model, whereas real firms raise capital and pay for their debts, taxes, manufacturing and marketing expenses, etc. Any model neglecting debt redemption is inconsistent with business practice. Therefore, the GBM models, trying to walk around debt and tax payments with artificial debt financing schemes, must be verified with the precise models taking into account all payments and investments. We present such a model (the Extended Merton model, EMM) in Section 3. In the times of turbulent economy, when management errors and the losses they cause can be critical for the firm's survival, it is important to use precise models for planning the firm's development and the capital structure securing it. Section 2 presents a critical analysis of the theoretical underpinning of the trade-off theory. Section 4 demonstrates modeling results of an all-equity firm and the identical levered firm. Section 5 concludes the paper.

UNDERPINNINGS OF THE TRADE-OFF THEORY

Modigliani-Miller Propositions

Modigliani and Miller (1958) consider firms in the perfect market described with the assumptions:

- (1) The firm value is determined only by the mean cash flow generated by the firm;
- (2) All investors have full information about the firms' cash flows;
- (3) There are an 'atomistic' competition and no market friction of any kind. At the market of corporate stocks and bonds, (a) there are no agency costs, (b) bankruptcy entails no liquidation costs, and (c) all investors, both individuals and institutions, can borrow at the same rate as corporations;
- (4) The debt of firms and investors is riskless;

- (5) There are no corporate or personal income taxes;
- (6) 'All firms can be divided into "equivalent return classes" (Miller (1988) calls them 'risk classes' - VS) such that the return on the shares issued by any firm in any given class is proportional to (and hence perfectly correlated with) the return on shares issued by any other firm in the same class. ... All relevant properties of a share are uniquely characterized by specifying (1) the class to which it belongs and (2) its expected return (Modigliani & Miller 1958, p. 266).' The authors claim that this assumption permits them 'to classify firms into groups within which the shares of different firms are "homogeneous," that is, perfect substitutes for one another (ibid).'

We will show that assumptions (1) and (6) are equivalent to the statement that neither the levered firm nor the unlevered one makes any payments. Assumption (1) contradicts Markowitz's mean-variance theory of efficient portfolio allocation (Markowitz 1952), who assumes that the firm value is determined by its mean cash flow and risk represented by the cash flow variance. It seems natural to characterize the risk class with the expected return and variance in the MMP problem setting. There is also a hypothesis that the higher the firm's debt, the greater the risk of default. Discussing the effect of debt on the value of the firm's warrant, Merton (1973, p. 151) says, 'If the firm changed its capital structure by raising the debt/equity ratio, then the riskiness of the common stock would increase, and the warrant would become more valuable.' Leland (1994) uses this hypothesis as an established fact to prove that the firm's return is a convex-up function of debt leverage. Using Markowitz's hypothesis and the dependence of the firm risk/variance on its debt leverage, one can conclude that the unlevered firm and the identical levered firm can never get into the same risk class. So, the firm value depends on its capital structure, contrary to the MMP1. We see that the MMP1 and Markowitz's theory are mutually exclusive, but they have been equally cordially accepted by the economists worldwide and rewarded with the Nobel Prizes.

However, the idea that debt automatically increases the firm's risks leads to an unsolvable problem. How does the firm distinguish between the debt dollar and the firm's own dollar to take into account that the debt dollar has a higher return variance? Obviously, those two dollars are absolutely identical to the firm at the start of its business activities. Their difference reveals itself at the moment of debt redemption: the unlevered firm has nothing to pay, whereas the levered firm pays for the debt to its creditors. The EMM shows that the more the firm pays, the higher the risk of default. None of the contemporary risk models can prove this dependence because, using the GBM equation, they exclude investments and payments from consideration.

To prove MMP1, the authors consider two firms in the same risk class, an unlevered firm and the levered one identical to the first in all respects but the capital structure. If the value of the levered firm is higher than that of the unlevered firm, then investors can sell shares of the levered firm, buy cheaper shares of the unlevered one, and, borrowing enough debt, reproduce the capital structure of the levered firm in their portfolios at a lower cost. As a result, the price of the overvalued shares will fall, and the price of the undervalued shares will rise until both prices are equal. The authors conclude that the capital structure does not affect the firm equilibrium value. However, there are two other possibilities in this line of reasoning: (1) an unlevered firm and the identical levered firm

can never get into the same risk class (because of different variances when the risk class is defined by the mean return and variance, Markowitz 1952), and (2) the risk classes with the properties required by Assumptions (1) and (6) do not exist.

We will not follow the first conclusion because it leads to the logical dead end; see the explanation above. Besides, Markowitz's firms, whose value is determined by the mean return and variance, do not make investments or payments, see the explanation below. To prove the conclusion (2), one has to show that the firms in the MMPs make no payments and their assets follow the GBM equation. First, we make a natural assumption that the firm value follows a continuous random walk. According to the MMP's Assumption (1), each firm in a risk class has a constant mean return. The assumption that the mean return remains constant means that the firm value development is a martingale process (Harrison & Kreps 1979), generated by the GBM equation (Merton 1974): $dS/S = \mu dt + \sigma dW$ (see comments to Equation (4)). To have martingale returns, the firm must be equivalent to a self-financing portfolio, where all portfolio returns go to the future investments only; there is no external inflow to and no outflow from that portfolio (Harrison & Kreps 1979). Thereby, any firm in a risk class makes neither payments nor investments and can be described by the GBM equation. Of course, Modigliani and Miller were not aware of the martingale argument in 1958, because it appears only in the 1980s.

The fact that MMP firms make no payments makes the MMPs obvious. Because there are no debt payments, the capital structure has no effect on the firm value (MMP1 1958). Because the levered firm does not pay for its debt but enjoys the tax shield, its mean after-tax value is higher than the mean after-tax value of the unlevered firm due to tax deductions (MMP3 1963). Here MMP3 commits a logical error. To enjoy tax deductions, the levered firm, while making no debt payments, must receive tax deductions for the nominal debt. In other words, the levered firm makes arbitrage profits the higher, the greater the debt. To resolve this arbitrage problem, one must note that the levered firm cannot be distinguished from the identical unlevered firm because the levered firm does not pay for its debt, and, therefore, there are no tax deductions for this firm. So, in the perfect market with corporate taxes, the after-tax value of the levered firm is *not greater but equal* to the after-tax value of the unlevered firm. This fact eliminates the very possibility of the tax gains for the levered firm promised by the MMP3. If one takes account of payments, even in the proportional form, $P = \alpha S$, then neither MMP1 nor MMP3 holds.

Now we show that the MMP risk classes cannot exist for long because the levered and unlevered firms, starting in the same risk class, diverge from each other over time, ceasing the existence of that risk class (the MMP risk classes are unstable). We test the MMP1, using the original Merton model (1974):

$$dX = (\mu X - P)dt + CXdW, \quad X(0) = X_0, \quad (1)$$

here $X(t)$ is the firm's asset at time t , μ is the rate of instantaneous expected returns on the firm per unit time, P is the total dollar payouts by the firm per unit time to either its shareholders or liabilities-holders per unit time, constant C^2 is the variance of returns, and W is a Wiener process.

Let the unlevered firm make no payments, $\delta_U = 0$, and the levered firm make debt payments proportional to the firm assets, $P = \delta_L X$, $0 < \delta_L < \mu$. The firms are identical in all other respects: they have the same initial values X_0 , expected returns μ , and the same

intensity of shocks C . Equation (1), describing dynamics of both firms, becomes the GBM equation:

$$dX/X = (\mu - \delta)dt + CdW, \quad X(0) = X_0 \quad (2)$$

For the logarithmic variable $z = \ln(X/X_0)$, Equation (2) by Ito's lemma transforms to:

$$dz = Rdt + CdW, \quad z(0) = 0, \quad R = \mu - 0.5C^2, \quad \text{for the unlevered firm (3a)}$$

$$dz = (R - \delta_L)dt + CdW, \quad z(0) = 0, \quad \text{for the levered firm.} \quad (3b)$$

The mean return of the unlevered firm grows with rate R : $\langle z_U \rangle \equiv H_U(t) = Rt$, while the mean return of the levered firm grows with rate $R - \delta$: $\langle z_L \rangle \equiv H_L(t) = (R - \delta_L)t$. For $t > 0$, one has $H_U(t) > H_L(t)$, and the firm value depends on its capital structure in the GBM model with proportional payments.

To estimate the effect of taxes on the values of two identical levered and unlevered firms, we suppose that payments of the unlevered firm consist of dividends and taxes only, and the dividends are proportional to the firm value, $P_U = \text{DIV} = \delta_U X$, $0 < \delta_U < \mu$ (we take account of the taxes later), while payments of the levered firm consist of dividends, debt payments, and taxes $P_L = \text{DIV} + \text{DP} = \delta_L X$, $0 < \delta_U < \delta_L < \mu$, $P_L > P_U$. The returns $H_L(t)$ and $H_U(t)$ of both firms can be effectively considered as the returns of two unlevered firms with no payments but with different effective rates of return (see Equation 3). Correspondently, the returns of the levered firm are less than the returns of the unlevered firm. As effectively unlevered firms, both firms have no rights to the tax shield and pay their taxes at the same rate. Therefore, the after-tax mean value of the levered firm is less than the after-tax mean value of the unlevered one. This result refutes the MMP3 (1963). We see that debt negatively affects the after-tax mean value in the GBM model with proportional payments.

For the firms with payments, MMPs are wrong, and the value of the levered firm is always lesser than the value of the unlevered firm, provided all other parameters equal (see Sections 3 and 4). It is interesting that in his Nobel lecture, Miller (1990) angrily rejects the charge that the MMPs have provoked a wave of leveraged buyouts (LBOs), which hit hard at the business world in the 1980s. He argues that LBOs existed long before publication of the MMPs (1958, 1963), and LBOs of the 1980s differ only by a larger scale. He insists that the state of the firm is invariant to the capital structure in the absence of taxes, and the firm can have substantial gains from leveraging when taxes are included. These 'gains of the tax-driven kind have undoubtedly figured both in the rise of corporate debt ratios generally in the 1980s and in some recent LBOs' (Miller 1990, p. 293). So, he admits that the MMPs encouraged capital borrowing, promising tax gains. However, we have shown that the MMPs are wrong; actually, debt weakens the firm (see Sections 3 and 4), provoking LBO. Thus, Modigliani and Miller, together with the prodigious number of economists sharing their views, are morally responsible for their dubious contribution to the economic theory and misleading recommendations to business.

Although the MMPs are false, they have set up the basis for capital structure theories and determined their development for decades. No wonder that all cornerstone articles investigating financial risks, such as Black and Scholes (1973), Merton (1974), Leland (1994), and many others use the MMPs as the absolute truth, with sad consequences for their results and the financial economics theory and practice.

Capital Structure Theories in the 1980s

One can find an early history of the capital structure development after MMP publications in a literature review by Harris and Raviv (1991) embracing more than 150 theoretical articles. It surveys the capital structure theories based on agency costs, asymmetric information, and corporate control considerations (except the tax-based theories). Jensen and Meckling (1976) construct one of the first models in which capital structure is determined by the agency costs generated by the conflict of interests between shareholders and managers. The authors argue that having a greater level of debt in the firm's capital structure, while keeping constant the manager's absolute investment in the firm, mitigates the loss caused by the conflict between the managers and shareholders. Jensen (1986) points out that because debt commits the firm to pay out cash, it reduces the amount of 'free' cash available to managers to engage in pursuits of transferring the firm's resources to their personal benefits. The theory claims that this mitigation of the conflict between managers and shareholders constitutes the benefit of debt financing. Grossman and Hart (1982) show another benefit of debt financing. If bankruptcy is costly for managers because they lose the benefits of control or reputation, then debt can create an incentive for managers to work harder, make better investment decisions, consume fewer perquisites, etc., because this behavior reduces the probability of default. A somewhat naïve attempt to reduce the capital structure problem to the conflict of interests between shareholders and managers makes the agency theory more qualitative than quantitative, which is not enough for optimizing debt leverage.

Leland's Optimal Capital Structure Theory

Discussing the theory of capital structure and the cumulative effect of debt and tax on the firm value, one cannot pass by the celebrated article by Leland (1994) that won the first-ever Stephen A. Ross Prize (Newsweek 2008). The prize committee notes that Leland's significant contribution to capital structure research lies in the fact that he has developed an analytical model that incorporates both the tax benefits and bankruptcy costs associated with debt issuance. The committee also notes that the paper has had a substantial influence on subsequent research on capital structure and debt valuation. Following the line of research of Brennan and Schwartz (1978) and Black and Cox (1976), Leland presents a solution of the optimal capital structure problem. The author turns the hypothesis of Kraus and Litzenberger (1973) into a quantitative theory of capital structure.

Leland uses the standard assumptions of the perfect market plus the following four assumptions:

- (1) The Modigliani-Miller (1958) theorem of the capital structure irrelevance is obtained.
- (2) The dynamic of the productive assets of the firm, V , is described by a diffusion process with the GBM equation (Merton 1974):

$$dV/V = \mu dt + \sigma dW, V(0) = V_0 \quad (4)$$

where μ is the instantaneous expected rate of return on the firm per unit time, σ^2 is the instantaneous variance rate of returns on the firm per unit time, and W is a Wiener process.

- (3) There is also a claim on the firm, $F(V, t)$, that continuously pays a nonnegative coupon, C , per instant of time when the firm is solvent. When the firm finances the net cost of the coupon by issuing additional equity, this claim meets the equation (Black & Cox 1976):

$$F_t + rVF_V + 0.5\sigma^2V^2F_{VV} - rF + C = 0, \quad (5)$$

with boundary conditions determined by payments at maturity, or by payments in bankruptcy should this happen prior to maturity.

- (4) Capital structure decisions, once made, remain static.

Assumptions (1) and (2) are equivalent; see the discussion on the MMP1 and MMP3 above.

Leland, after Black and Cox (1976), looks for the time-independent solution, reducing Equation (5) to its stationary form:

$$rVF_V + 0.5\sigma^2V^2F_{VV} - rF + C = 0, \quad (6)$$

then, using its general solution (Black & Cox 1976)

$$F(V) = A_0 + A_1V + A_2V^{-X}, \quad X = 2r/\sigma^2, \quad (7)$$

he determines constants A_0 , A_1 , A_2 from boundary conditions at the infinity and at bankruptcy. Leland derives solutions for the firm's debt $D(V, C)$, bankruptcy costs $BC(V)$, tax benefits $TB(V, C)$, and the firm value $v(V, C)$:

$$D(V, C) = C/r + [(1 - \alpha)V_B - C/r](V/V_B)^{-X}, \quad (8)$$

$$BC(V) = \alpha V_B (V/V_B)^{-X},$$

$$TB(V, C) = \tau C/r [1 - (V/V_B)^{-X}],$$

$$v(V, C) = V + TB(V, C) - BC(V), \quad (9)$$

here α is the fraction of the firm value which is lost to bankruptcy and τ is the tax rate. Equations (8)-(9) make a quantitative basis of the trade-off theory, which Leland applies for finding the optimal capital structure for the exogenous default. The equation for $TB(V, C)$ represents the tax returns for the nominal debt. Equation (9) reveals that a third party pays for the claim C ; otherwise, this equation would include cumulative expenses for the coupon payments (infinite for the infinite maturity time). However, when the firm does not pay for its debt but receives tax returns, the firm makes arbitrage profits. This fact explains Leland's result: the greater the debt, the greater the firm value (in full agreement with the MMP3 discussed above). It is the bankruptcy costs that make the firm value a convex-up function of debt leverage, securing the existence of the optimal debt ratio. The arbitrage catastrophe voids all Leland's results. Another error committed by Leland is in using the marginal solution when time tends to infinity, whereas the diffusion problem (4)-(5) has no stationary solution. One can see that the mean asset tends to infinity: $\langle V(t) \rangle \sim \exp(\mu t + 0.5\sigma^2 t) \rightarrow \infty$ as $t \rightarrow \infty$, confirming that the stationary solution does not exist. As a consequence, the formulas (6)-(9) make no sense.

Leland and Toft (1996) revisit the problem of optimal capital structure with a debt of fixed maturity. The authors describe the firm's productive asset, V , by a continuous-time diffusion process with constant proportional volatility σ :

$$dV/V = [\mu(V, t) - \delta]dt + \sigma dW, \quad (10)$$

where $\mu(V, t)$ is the total expected rate of return on asset value V , δ is the constant fraction of the value paid out to security holders, and W is a standard Wiener process. The process continues until V falls to default triggering value V_B . The authors consider the bond issue with maturity t from the present, which has the principal $p(t)$ and continuously pays a constant coupon flow $c(t)$. In the event of bankruptcy, the debt of maturity t receives the fraction $\rho(t)$ of asset value V_B . Introducing the density of the first passage time τ from V to V_B , $f(\tau; V, V_B)$, and its cumulative distribution $F(\tau; V, V_B)$, and using the risk-neutral technique completely equivalent to the GBM equation, the authors derive a formula for the debt value of maturity t , $d(V; V_B, t)$:

$$d(V; V_B, t) = \frac{c(t)}{r} + e^{-rt} \left[p(t) - \frac{c(t)}{r} \right] [1 - F(t)] + \left[\rho(t)V_B - \frac{c(t)}{r} \right] G(t) \quad (11)$$

$$F(t) = N[h_1(t)] + \left(\frac{V}{V_B}\right)^{-2a} N[h_2(t)], \quad G(t) = \left(\frac{V}{V_B}\right)^{-a+z} N[q_1(t)] + \left(\frac{V}{V_B}\right)^{-a-z} N[q_2(t)],$$

where $N(\cdot)$ is the cumulative standard normal distribution and

$$h_1(t) = \frac{-b-a\sigma^2 t}{\sigma\sqrt{t}}, \quad h_2(t) = \frac{-b+a\sigma^2 t}{\sigma\sqrt{t}}, \quad q_1(t) = \frac{-b-z\sigma^2 t}{\sigma\sqrt{t}}, \quad q_2(t) = \frac{-b+z\sigma^2 t}{\sigma\sqrt{t}}, \quad (12)$$

$$a = (r - \delta - 0.5\sigma^2)\sigma^{-2}, \quad b = \ln(V/V_B), \quad z = (a^2\sigma^4 + 2r\sigma^2)^{1/2}\sigma^{-2}.$$

Tending maturity t to the infinity and setting $x = a + z$ in (11), the authors get

$$d(V; V_B, \infty) = \frac{c(\infty)}{r} \left[1 - \left(\frac{V}{V_B}\right)^{-x} \right] + \rho(\infty)V_B \left(\frac{V}{V_B}\right)^{-x}. \quad (13)$$

The authors state (p. 992) that Equation (13) is identical to Equation (8) of (Leland 1994) when $t \rightarrow \infty$ and $\rho(\infty) = 1 - \alpha$, proving the consistency of Leland's (1994) and Leland and Toft's (1996) results. However, Equation (8) has no payments financing the coupon flow, and its rate of return is r (see Equation 5), while Equation (13) includes proportional payments financing the coupon flow and has a decreased rate of return $r - \delta$ (see Equation 12 and definitions of a , z , and x). For proportional payments, the firm with payments is equivalent to the unlevered firm without payments but with a corrected rate of return; see Equation (3) above. Then the authors derive an equation for the firm value, including the assets, tax benefits, and bankruptcy costs, similar to Equation (9). Again, the tax benefits $TB(V, C)$ are the tax returns on the nominal debt, and, adding them to the asset value of the effectively unlevered firm, Leland and Toft make the firm a generator of arbitrage profits. The arbitrage catastrophe voids their results. We must remind the reader that the authors write their equation for the firm value in the stationary case when $t \rightarrow \infty$, while the diffusion problem (Equation 10) has no stationary solution. Therefore, all results derived within this framework have very little to do with the real firms paying their debts and taxes.

Another example of the phantom phenomenon caused by misunderstanding of the debt and tax influence on the firm value is the leverage ratchet effect reported by Admati, DeMarzo, Hellwig, & Pfleiderer (2018). The authors describe this effect as 'the (shareholder) resistance to leverage reductions, together with the desire to increase leverage, creates asymmetric forces in leverage adjustments (the leverage ratchet effect).' At that, shareholders make the firm issue the additional debt in sequential rounds until all the tax advantages of debt are exhausted. In other words, Admati and his co-authors builds the leverage ratchet effect on the positive influence of debt on the firm value claimed by MMP3

(1963). The authors develop a sophisticated risk-neutral analysis of the leverage choices of a firm with outstanding debt already in place. We remind the reader that the risk-neutral approach is equivalent to the analysis based on the GBM equation that includes neither investments nor payments and, therefore, it cannot be applied to consideration of the firms with significant levels of debt. The incorrect initial idea and the inadequate method of analysis have brought the authors to their false conclusions.

As we have shown, the classic papers by Modigliani and Miller (1958, 1963), Leland (1994), and Leland and Toft (1996), which make the theoretical underpinning of the trade-off model of capital structure, are wrong. Their failure stems from using the GBM equation, which does not consider the effects of the firm's payments and investments. Any model neglecting investments and payments cannot be true because it contradicts the behavior of real-life firms. Unfortunately, most modern models of capital structure use the GBM equation (e.g., Mella-Barral & Perraudin 1997, Goldstein et al. 2000, Hackbarth et al. 2006, Strebulaev 2007, Diamond & He 2014, Leland & Hackbarth 2019, Chen et al. 2022). Therefore, their conclusions must be verified with the precise models, taking into account the firm's payments and investments. We present such a model (the EMM) in Section 3. The EMM shows that debt deteriorates the firm dynamic and decreases its value compared to the dynamic and value of the identical unlevered firm; the greater the debt, the faster the firm value falls. The firm value as a function of debt leverage has no maximum; it is a monotonously decreasing function of debt. The optimal leverage and the maximum firm value it allegedly provides are just the phantoms caused by errors in (Leland 1994) and (Leland & Toft 1996). The papers, insisting on the beneficial influence of debt on the firm value and the optimal capital structure in the spirit of MMPs and Leland, make no sense. However, for years and years since 1973, when Kraus and Litzenberger put forward their capital structure hypothesis, this idea has seized the minds of academics and business managers and remains popular up to now. The trade-off and pecking-order models dominate the capital structure literature and business practice because they suggest simple rules for choosing capital structure.

Empirical Studies of the Trade-off Theory

The trade-off theory has four empirically testable consequences. First, each firm must have a target debt ratio determined by parameters of the firm and its business environment. When the firm debt ratio deviates from the target, the firm must adjust it towards the target ratio (Myers 1984). Second, the higher the firm risk, the lesser the debt ratio must be (Myers 1984). Particularly, firms with tangible assets are expected to have more debt than firms with more risky intangible assets. Third, firms with higher tax rates must have higher debt ratios (MacKie-Mason 1990). Fourth, firms with more taxable income and few nondebt tax shields (investment tax credits, depreciation, etc.) must have higher debt ratios (DeAngelo & Masulis 1980). However, numerous attempts to provide empirical support for these predictions or to reject them meet with mixed success.

Graham and Leary (2011) group explanations of the inadequate performance of the traditional models into the following categories: (i) important dependent and explanatory variables have been mismeasured; (ii) the impact of leverage on nonfinancial stakeholders is important; (iii) the supply side of capital affects the corporate capital structure; (iv) richer features of financial contracts should be considered and should add new dimensions

to study; (v) value effects due to capital structure variation are modest over wide ranges of leverage; (vi) estimates of leverage adjustment speeds are biased; (vii) the implications of capital structure dynamics have not been adequately considered. At that, the authors accept the logic of the trade-off model based on the MMPs (1958, 1963) and Leland (1994) as flawless. As a consequence, they missed maybe the most important category: empirical studies guided by a false theory.

One can find the leverage patterns consistent with the trade-off logic in the articles by Harris and Raviv (1991), Rajan and Zingales (1995), Fama and French (2002), and Frank and Goyal (2008), but there are also serious deviations from that logic. First, a negative relation is documented between profitability and leverage, inconsistent with the trade-off model (Eckbo & Kisser 2021). There are also firms with very low (or zero) leverage (Graham 2000). Second, the trade-off predictions explain only a relatively small part of the observed capital structure variations (Lemmon et al. 2008). Third, research suggests that the rate of reversion to target is too slow to be considered as a first-order policy determinant (e.g., Fama & French 2002, Baker & Wurgler 2002, Welch 2004, Eckbo & Kisser 2021). Fourth, the expected cost of distress appears to be much smaller than the corporate tax benefits that could be achieved by using more debt (Graham 2000). Willing to raise the expected cost of distress, Almeida and Philippon (2007) suggest estimating the cost of distress using the risk-neutral, instead of physical, default probabilities (note that risk-neutral probabilities exist only for the firms with no payments). Graham and Leary (2011) provide a detailed description of inconsistencies between the trade-off theory and the capital structure behavior of real-life firms.

Eckbo and Kisser (2021) statistically reject the core predictions of the trade-off theory. They identify and analyze the leverage dynamics of high-frequency net-debt issuers (HFIs). HFIs are public industrial companies raising bulk of all private and public debts. The combination of high debt-financing benefits and low issuance costs make HFIs most flexible about capital structure decisions and, thus, good objects for testing the trade-off theory. The authors examine three core trade-off predictions which so far have received mixed empirical support. Their first hypothesis is that more profitable firms will move to the target leverage when issuing debt. The authors report that their cross-sectional test firmly rejects this hypothesis. The second hypothesis is that the speed-of-adjustment of debt deviations to the target debt leverage is greater for HFIs than for low-frequency net-debt issuers. Under the dynamic trade-off theory, greater issue benefits net of issue costs drives higher debt-issuance frequencies. However, the test has shown that the speed-of-adjustment estimates are statistically indistinguishable across these two groups of firms. The third hypothesis relates investment decisions and the leverage dynamics of HFIs. The first question here is whether the speed-of-adjustment increases in periods of low leverage and high investments. The authors find no evidence for the positive answer to this question. The second question is whether debt issues by already over-leveraged firms are repurchased when the investment-financing need abates. The authors find no evidence that these debt-issues are followed by active debt repurchase in the three-year period following the year of debt issue. More than that, the authors show that the under-leveraged HFIs repurchase debt nearly as often as when they are over-leveraged. The objective of these actions is very likely to preserve financing flexibility rather than to manage leverage towards a target. Eckbo and Kisser (2021) make a weighty empirical contribution to the exposure of the trade-off inconsistency.

Our explanation of the mixed success in the attempts to support or reject the trade-off model is as follows. The trade-off model, together with the pecking-order model (Myers & Majluf 1984), is one of two dominating theories of capital structure, suggesting simple and direct instructions on how to choose the debt ratio. Unlike the pecking order, the trade-off model promises the tax shield gains that peak at the optimal leverage. The model has been theoretically founded by Modigliani and Miller and has withstood thirty years of criticism from the world economic community; so, practical managers take this model for its face value and introduce it extensively to their practice. However, its practical results appear to be doubtful, and many managers begin to use the trade-off model together with the pecking-order model or various heuristic methods in attempts to improve the capital structure results. The estimation of the trade-off efficiency is complicated by the stochastic nature of the problem and its variables: the firm value, the firm's returns, etc. Practical results of the trade-off model are contradictory because this model is false, and the effects it promises do not exist, but the researchers anticipate them. All that makes the empirical studies of the trade-off theory very inconsistent, and the capital structure literature documents it.

DeAngelo (2022), after Graham and Leary (2011), notes that 60-plus years since publication of the MMP1 (1958), 'we still do not have a model that explains even the broad-brush features of observed capital structures. Nor have we had much success in descriptive work, with a wide-ranging set of studies struggling to pin down significant empirical drivers of capital structure (p. 413).' The author regrets that 60-plus years of research using state-of-the-art empirical methods and highly sophisticated theoretical analysis have given so little for the capital structure theory. Answering this regret, we can say that empirical work can be fruitful only if it is carried out within the framework of a correct hypothesis or theoretical model. However, we have shown that highly sophisticated theoretical models of the firm, based on Merton's GBM equation, are wrong because they neglect the firm's payments and investments. That is why state-of-the-art empirical methods guided by false theories provide such vague results.

DeAngelo (2022) rejects the standard trade-off models, which fail empirically because they ignore funding and focus on optimizing the debt-equity mix. These models ignore capital infusions over time, connections among security issuances, cash build-ups/draw-downs, etc. Showing his full trust in the MMPs (1958, 1963), DeAngelo believes that the standard trade-off 'models are best reserved for understanding firms at a stage of lifecycle where funding is not important (p. 418).' The author formulates a set of principles that an empirically credible model of capital structure must follow: (i) emphasize reliable access to funding; (ii) recognize that financial policies are not pinned down uniquely by economic fundamentals; (iii) include incentives for a connection between debt issuance and investment; (iv) incorporate a role for firms using and replenishing debt capacity and cash balances to meet future funding needs; (v) accommodate managers acting on a belief they can time the capital markets; (vi) include a role for financial intermediation in providing reliable access to funding for operating firms. For the case when the firm's funding comes from a bank, the author demonstrates how his qualitative baseline model explains various phenomena in capital structuring unsolvable for other capital structure models. Principle (iv) makes firms discharge their debt rather fast to prepare debt capacity and cash balances to meet future funding needs, unlike the trade-off model, which insists on maintaining the optimal debt leverage. DeAngelo claims that managers cannot optimize capital structure

because there is no hard relation between the firm's economic fundamentals and its financial policy. He insists that experimentation is the natural path for managers who seek better financial policies for their firms. However, one can hardly believe that trial and error experimentation makes the best financial management for real firms. A mathematical model adequately mapping funding operations and the firm's payments could be a useful instrument in analysis of numerous scenarios of choosing the best capital structure as a function of time and the environment and firm's parameters. The EMM is exactly such a model.

Beattie, Goodacre, and Thompson (2006) show the incidence of the trade-off theory as guidance for choosing capital structure in business practice. In a comprehensive survey of the corporate financing decision-making process in UK companies, the authors present a comparison between practice and extant theories of capital structure. They reveal how companies determine their overall financing strategy and why they pick up a particular mix of financing instruments. One of their key findings is that half of the firms (51%) seek to maintain a target debt level consistent with the trade-off theory, while 59% of responding firms report that they follow the pecking-order theory (Myers & Majluf 1984). This fact reflects the dominating position of the trade-off and pecking order among all other theories of capital structure. At that, 32% of firms claim that they follow both theories, and 22% follow neither of them. It means that the respondents do not consider these two theories as either mutually exclusive or exhaustive. The firms using the trade-off theory have debt targets ranging from 0% to 300% with a median of 40%, and with 80% of all companies indicating a target debt level of 50% or less. Graham and Harvey (2001) report that 37% of their respondents among U.S. firms have a flexible target, 34% have a somewhat tight target or range, and 10% have a strict target.

The surveys by Beattie, Goodacre, and Thompson (2006) and Graham and Harvey (2001) document that a significant part of the responding firms follow the false trade-off theory when choosing their capital structure. At present, the firm's management has no methods of estimating the firm value dynamic and the firm's risks in the five-year or longer periods, while the destructive effect of debt develops slowly at first and gains momentum with each next year, threatening the firm with heavy losses up to default. To the best of our knowledge, existing models make their forecast for one year only (e.g., CreditMetrics from JP Morgan, Moody's KMV, and CreditRisk+ from Credit and Risk Financial Products) and cannot disclose a full picture of debt effects. It could be very dangerous for a firm to take a heavy debt in times of the turbulent world economy like that we have now.

THE EXTENDED MERTON MODEL (EMM)

The Extended Merton model (EMM; Shemetov 2020, 2023) starts with the continuous-time equation describing the firm value $X(t)$ in a stochastic environment (Merton 1974):

$$dX = (\mu X - P)dt + CXdW, \quad X(0) = X_0, \quad (14)$$

here constant μ is an instantaneous expected return on the firm per unit time, P is total dollar payouts by the firm per unit time, constant C^2 is the variance of returns, W is a Wiener process. The firm makes various payments while doing its business. Some of them are for the manufacturing and marketing of the firm's goods; these payments are considered as continuous-time payments (CP). Other payments secure the firm's presence in business

(the business securing expenses, BSEs). Such expenses include fixed costs (FC), debt payments (DP), and taxes (TAX); these payments are made by installments at the end of a time unit (e.g., a year). One can write for the firm's payments P :

$$P = CP + BSE, BSE = FC + DP + TAX, \quad (15)$$

$$CP(t) = P_0\pi(t), CP(0) = P_0, \pi(0) = 1, P_0 > 0.$$

The function $\pi(t)$ is a piecewise continuous function of time. Equation (14) for the random variable $x = \ln(RX/P_0)$ by Ito's Lemma transforms to

$$dx = R(1 - \pi(t)e^{-x})dt + CdW, \quad (16)$$

$$x(0) = x_0 = \ln\left(\frac{RX_0}{P_0}\right), R = \mu - 0.5C^2.$$

Writing a Fokker-Plank equation for Equation (16), one comes to an equation for the probability distribution $V(x, t)$; V_y is a partial derivative over a variable y :

$$V_t + R(1 - \pi(t)e^{-x})V_x - 0.5C^2V_{xx} + R\pi(t)e^{-x}V = 0. \quad (17)$$

The initial condition for this problem is

$$V(x, 0) = V_0(x; H_0, \sigma_0^2), \quad (18)$$

where $V_0(x; H_0, \sigma_0^2)$ is a normal function with the mean H_0 and the variance σ_0^2 , and a boundary condition implying that the firm comes to default when its value falls to X_D is

$$V(DL, t) = 0, DL = \max\left[\ln\left(\frac{RX_D}{P_0}\right), 0\right]. \quad (19)$$

If $X_D(t)$ is an outstanding debt at time t as it is in (Black & Cox 1976), Equation (19) is an *exogenous* constraint. If the firm has no debt, the line $x = 0$ makes a soft *endogenous* boundary that separates business profits from losses. Below the line $x = 0$, the firm can continue its activities only selling its assets. A solution of the problem (17)-(19) is the log-value distribution for the firm in financial distress; it is denoted as $\hat{V}(x, t)$. If one knows the solution $V(x, t)$ in the open space, then the solution $\hat{V}(x, t)$ of the boundary problem can be written as

$$\hat{V}(x, t) = V(x, t) - V(2DL - x, t). \quad (20)$$

The intensity of default probability $DPInt(t)$ is

$$DPInt(t) = 2 \int_{-\infty}^{DL} V(x, t)dx, \quad (21)$$

and the default probability of the firm over a chosen time interval (e.g. debt maturity):

$$DPr(t_s, \tau) = \int_{t_s}^{t_s+\tau} DPInt(t)dt. \quad (22)$$

The model computes statistical moments for the firm:

$$\begin{aligned} \hat{H}(t) &= \int_{DL}^{\infty} x \hat{V}(x, t)dx, \quad \widehat{VAR}(t) = \int_{DL}^{\infty} (x - \hat{H})^2 \hat{V}(x, t)dx, \\ \widehat{SK}(t) &= \int_{DL}^{\infty} (x - \hat{H})^3 \hat{V}(x, t)dx, \quad \widehat{KT}(t) = \int_{DL}^{\infty} (x - \hat{H})^4 \hat{V}(x, t)dx, \\ Skew(t) &= \widehat{SK} \times \widehat{VAR}^{-3/2}, \quad Kurt(t) = \widehat{KT} \times \widehat{VAR}^{-2} - 3, \\ \widehat{MX}(t) &= \frac{P_0}{R} \int_{DL}^{\infty} e^x \hat{V}(x, t)dx. \end{aligned} \quad (23)$$

Equations (17)-(19) describe the behavior of the firm's assets in the interval ($t_{k-1} \leq t < t_k$) when the firm makes only continuous payments. In the end of the year $k = 1, 2, \dots, t = t_k$, the unlevered firm pays its corporate income tax; the levered firm pays its obligations to debtholders (DP) and the corporate income tax (TAX). We will not consider dividend payments here.

The effect of one-time payment DP makes an instant left shift LDP of each point of the distribution (x, t_k):

$$LDP(x, t_k) = \ln\left(\frac{x-DP}{x}\right), \quad x > DP. \quad (24)$$

Payment DP makes the boundary separating the firms who survive that debt payment from those who default. The distribution function for the firms that have survived the debt payment is

$$\hat{V}(x, t) = V(x, t) - V(2BP - x, t), \quad x \geq BP, \quad BP = \ln\left(\frac{R \times DP}{P_0}\right), \quad (25)$$

and the transformed distribution of assets of the levered firm after the payment assumes the form

$$x^{(DP)} = x + \ln(1 - e^{BP-x}), \quad (26)$$

$$V_1(x^{(DP)}, t_k^+) = (1 - e^{BP-x})\hat{V}(x, t_k^-), \quad (27)$$

$$BP \leq x < \infty, \quad -\infty < x^{(DP)} < \infty.$$

Here t_k^- and t_k^+ are the moments in the end of t_k - year *before* and *after* DP payment. This transformation produces different effects on high ($x \gg BP$) and low ($x \cong BP$) x -values in the distribution: high x -values shift insignificantly, while low x -values travel large distances to the left developing negative distribution skewness. This skewness increases the default probability for all firms in the ensemble, and brings to immediate default the firms with $x \leq BP$.

For the unlevered firm, one has

$$V_1(x, t_k^-) = \hat{V}(x, t_k^-), \quad 0 \leq x < \infty, \quad (27a)$$

with the default line $DP = 0$; t_k^- is the moment before the tax payment.

The tax payments follow the rules:

- $TAX(X, t_k) = 0$, if the taxable year income $INCOME(X, t_k) \leq 0$
- $TAX(X, t_k) = tax \left(1 - \frac{X_D(t_k)}{X}\right) INCOME(X, t_k)$, if the taxable income $INCOME(X, t_k) > 0$,

here tax is the tax rate, and the term $X_D(t_k)/X$ represents the share of income protected by the tax shield. We introduce functions $b_k = b(t_k)$ and $F(x, t)$ as:

$$\frac{X_D(t_k)}{X} = \frac{RX_D(t_k)}{P_0} e^{-x} \equiv b_k e^{-x}, \quad INCOME(X, t_k) \equiv \frac{P_0}{R} e^x F(x, t_k) \quad (28)$$

Now $TAX(x, t_k) = 0$, if $F(x, t_k) \leq 0$,

and $TAX(x, t_k) = tax \frac{P_0}{R} (1 - b_k e^{-x}) e^x F(x, t_k)$, if $F(x, t_k) > 0$.

The tax payment causes an instant left shift of each point of the distribution $V_1(x, t_k)$:

$$\ln\left(\frac{x-TAX}{x}\right) = \ln[1 - tax(1 - b_k e^{-x})F(x, t_k)], \text{ if } F(x, t_k) > 0, \quad (29)$$

$$\ln\left(\frac{x-TAX}{x}\right) = 0, \text{ if } F(x, t_k) \leq 0.$$

Using formulas (29), one has for the new variable $x^{(TAX)}$ and the asset distribution after tax payment, $V_2(x^{(TAX)}, t_k^+)$

$$x^{(TAX)} = x + \ln[1 - tax(1 - b_k e^{-x})F(x, t_k)], \text{ if } F(x, t_k) > 0, \text{ else } x^{(TAX)} = x, \quad (30)$$

$$V_2(x^{(TAX)}, t_k^+) = \frac{1-tax(1-b_k e^{-x})F}{1-tax[F+(1-b_k e^{-x})F_x]} V_1(x^{(DP)}, t_k^-), \text{ if } F(x, t_k) > 0, \text{ else} \quad (31)$$

$$V_2(x^{(TAX)}, t_k^+) = V_1(x^{(DP)}, t_k^-),$$

$$F(x, t) = \int_{-\infty}^x (1 - e^{-(x-x')})g(x, t|x', t-1)V_2(x', t-1)dx'.$$

For the unlevered firm having no tax shield

$$x^{(TAX)} = x + \ln[1 - taxF(x, t_k)], \text{ if } F(x, t_k) > 0, \text{ else } x^{(TAX)} = x, \quad (32)$$

$$V_2(x^{(TAX)}, t_k^+) = \frac{1-taxF}{1-tax(F+F_x)} V_1(x, t_k^-), \text{ if } F(x, t_k) > 0, \text{ else} \quad (33)$$

$$V_2(x^{(TAX)}, t_k^+) = V_1(x, t_k^-).$$

Omitting in the after-tax distribution index (TAX), one can write the after-tax asset distribution as $V_2(x, t_k^+)$. This distribution is fed as an initial condition to the problem (17)-(19):

$$V(x, t_k) = V_2(x, t_k^+),$$

which then is solved in the next year interval $t_k \leq t < t_{k+1}$.

EXAMPLE OF CUMULATIVE EFFECT OF DEBT AND TAX ON FIRM VALUE

To illustrate the effects of debt and tax on the firm value in the EMM framework, we consider the case of two small- or medium-sized firms (SMEs). The practical significance of this case becomes clear if one recalls that US SMEs (a) make up 99.9% of all businesses, (b) provide jobs for about 50-60% of the private sector workforce, (c) can be credited with creating two out of every three new jobs in the US economy, and (d) drive innovation, competition, and overall dynamism in the national economy.

SMEs cannot borrow capital from the market, and when they need funding, they take bank loans. Unlike SMEs, big firms can collect the necessary capital from the market, issuing bonds. The firm dynamic with the market debt differs significantly from the firm dynamic with a bank loan; we will consider the case of public debt in a special paper.

Let there be a business project, promising an expected annual rate of return of μ percent on the firm's assets, to start which a firm must have assets of no less than 1,000 dollar units. Suppose also that a firm has assets $X_0 = 1,000$ and enters this project (the all-equity or unlevered firm). This firm pays its BSEs in the form of taxes; thus, for the unlevered firm the payments are $P_U = CP + TAX$, where $CP(t)$ is continuous-time payments, while taxes

are paid in a lump sum once a year. The rest of the firm parameters are $H_0 = 2.0$, $VAR_0 = 0.02$, $R = 0.10$, $MX_0 = 1,000$, $P_0 = 13.40$, $tax = 0.3$; see parameter definitions in Section 3.

The dynamic of the unlevered firm distribution is shown in Figures 1-4. A comparison of the initial asset distribution and the distribution at the end of the first year before tax payment is shown in Figure 1. One can see that earnings drive the asset distribution right. Figure 2 presents the asset distribution at the end of the first year *before* and *after* tax payment; the tax payment increases the distribution's right skewness. The evolution of asset distribution after tax payments over five years is given in Figure 4. It shows the distribution persistently shifts right, and the positive distribution skewness grows over time; the default probability remains very low. One can find the evolution of statistical moments in Table 1. These results prove a steady positive development of the unlevered firm. Table 1 shows numerical characteristics of this dynamic.

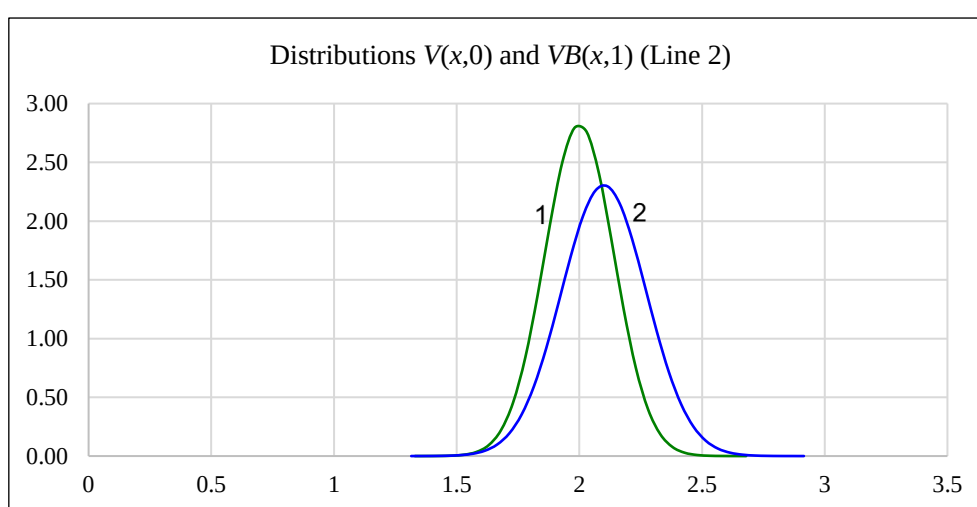


Figure 1: Log-asset distributions of unlevered firm at $t = 0$, $V(x, 0)$, and $t = 1$ before tax payment, $VB(x, 1)$ (Line 2).

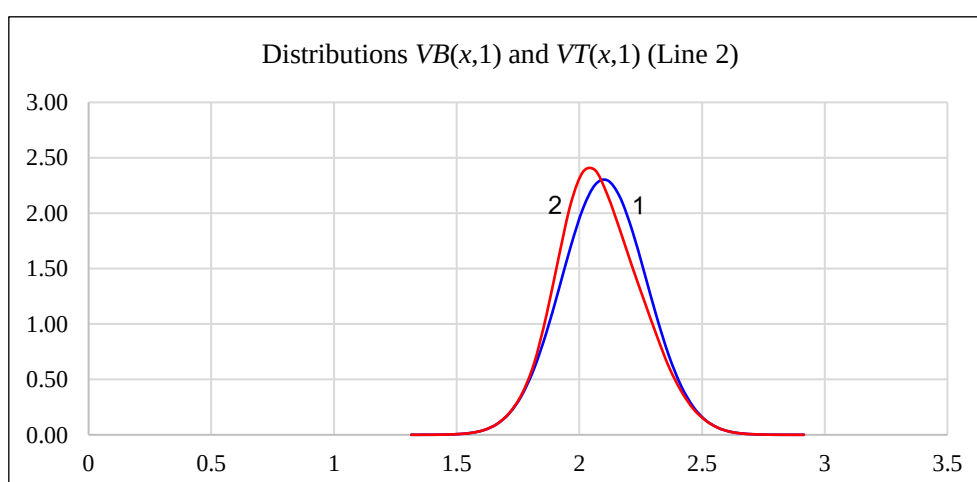


Figure 2: Log-asset distributions of unlevered firm at $t = 1$ before $VB(x, 1)$ and after $VT(x, 1)$ tax payment (Line 2).

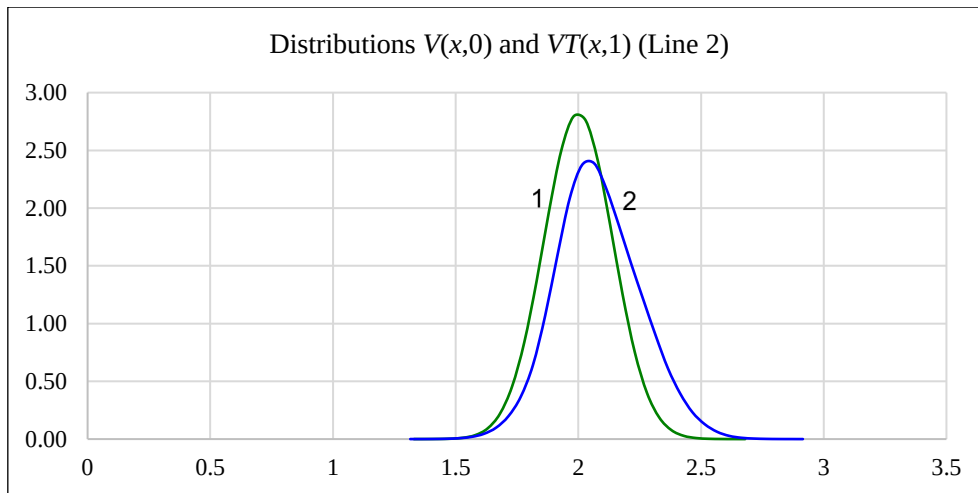


Figure 3: Log-asset distribution of the unlevered firm at $t = 0$, $V(x, 0)$, and $t = 1$ after tax payment, $VT(x, 1)$ (Line 2).

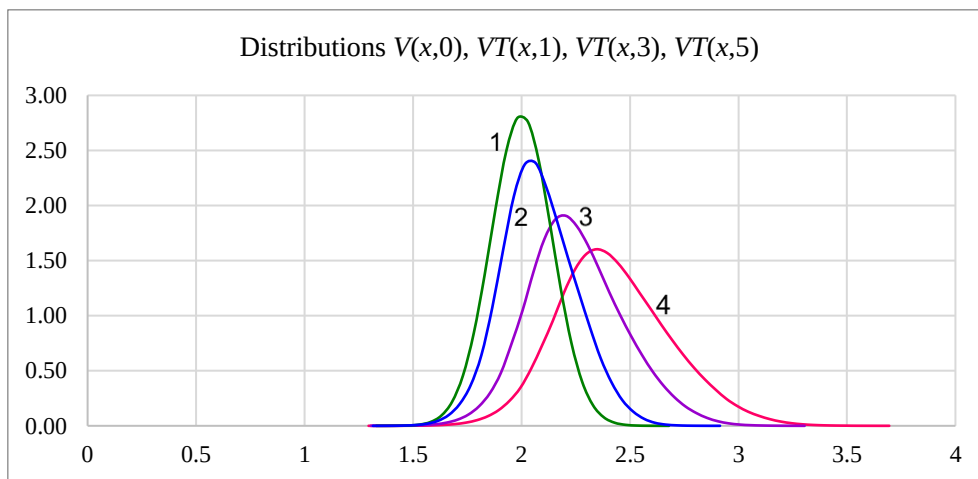


Figure 4: Evolution of log-asset distributions of unlevered firm $V(x, 0)$ (Line 1), $VT(x, 1)$, (Line 2), $VT(x, 3)$ (Line 3), $VT(x, 5)$, (Line 4) after tax payment.

Table 1: Evolution of Statistical Moments for the Asset Distribution of the Unlevered Firm After Tax Payments

Year	Mean return $H(t) - H_0$	Variance $VAR(t) - VAR_0$	Skewness	Kurtosis
0	0	0	0	0
1	0.081	0.0096	0.184	0.070
2	0.165	0.0191	0.252	0.126
3	0.247	0.0287	0.343	0.179
4	0.336	0.0386	0.352	0.165
5	0.422	0.0485	0.361	0.160

Now we consider another firm identical to the first one in all respects but the size of assets. Let the assets be $X_0 = 1,000$ - A dollar units. To enter the project, the firm takes a bank loan of A dollar units for T_m years at the interest rate of r percent (the levered firm).

The total debt makes $D(0) = A \exp(rT_m)$. Let the firm pay its debt once a year in equal installments: $DP = (A/T_m) \exp(rT_m)$. The levered firm pays its BSEs as debts and taxes; the total payment for the levered firm makes $P_L = CP + DP + TAX$. As before, CP is a continuous payment, and debts and taxes are paid in a lump sum once a year. The firm parameters are $H_0 = 2.0$, $VAR_0 = 0.02$, $R = 0.10$, $MX_0 = 1,000$ dollar units, $A = 100$ dollar units, $T_m = 5$ yrs, $r = 0.05$, the total debt is 128.4 dollar units, $P_0 = 13.40$, $tax = 0.3$. We trace down the firm evolution in the interval of 10 years.

The dynamic of the levered firm distribution is shown in Figures 5-11. Figure 5 presents changes in the asset distribution of the levered firm after year earnings; distribution $VB(x,1)$ shifts right about the initial distribution $V(x,0)$. Figure 5 for the levered firm and Figure 1 for the unlevered firm are identical. This identity reflects the obvious notion that the difference between the firm's own capital and debt reveals itself at the moment of debt redemption, not before.

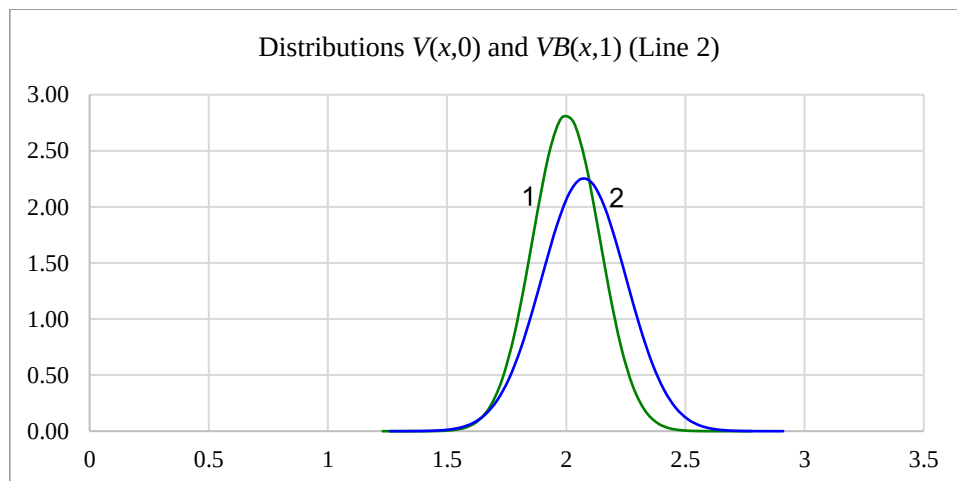


Figure 5: Log-asset distributions of levered firm at $t = 0$, $V(x, 0)$, Line 1, and $t = 1$ before debt payment, $VB(x, 1)$, Line 2.

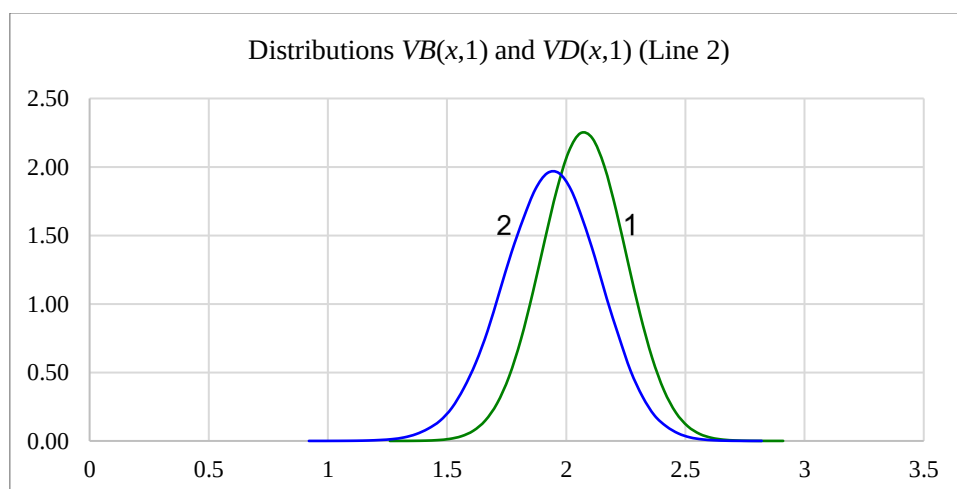


Figure 6: Log-asset distributions of levered firm at $t = 1$ before (Line 1, $VB(x, 1)$) and after debt payment (Line 2, $VD(x, 1)$).

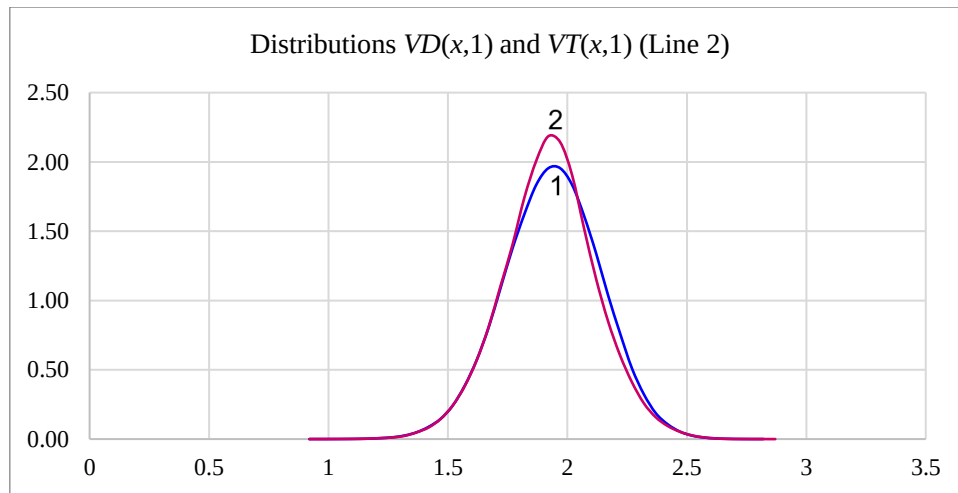


Figure 7: Log-asset distributions of levered firm at $t = 1$ before (Line 1, $VD(x, 1)$) and after tax payment (Line 2, $VT(x, 1)$).

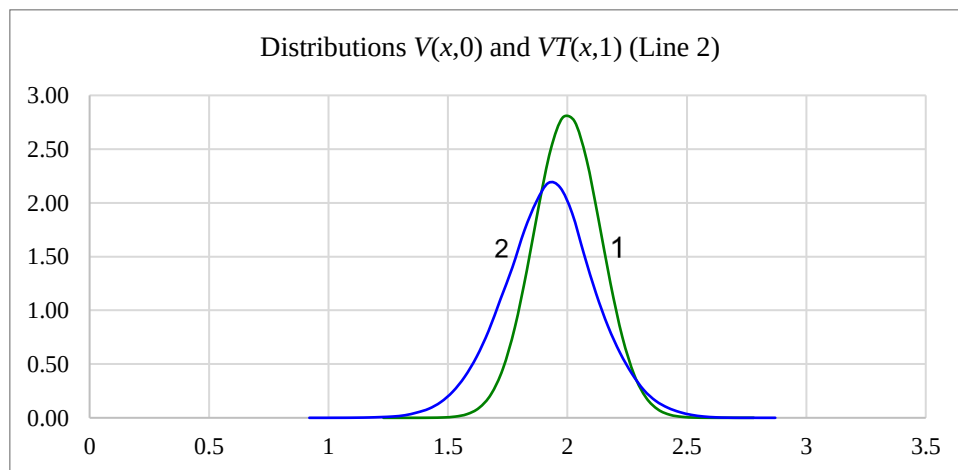


Figure 8: Log-asset distributions of levered firm at $t = 0$, $V(x, 0)$, Line 1, and $t = 1$ after debt and tax payments, $VT(x, 1)$, Line 2.

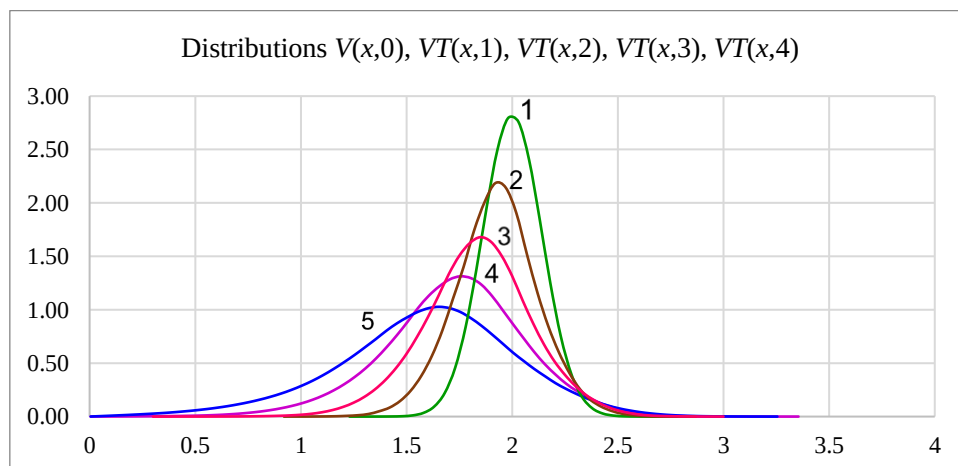


Figure 9: Log-asset distributions of levered firm at $t = 0$, $V(x, 0)$, Line 1, and $VT(x, t)$ at $t = 1$ (Line 2), 2 (Line 3), 3 (Line 4), 4 (Line 5).

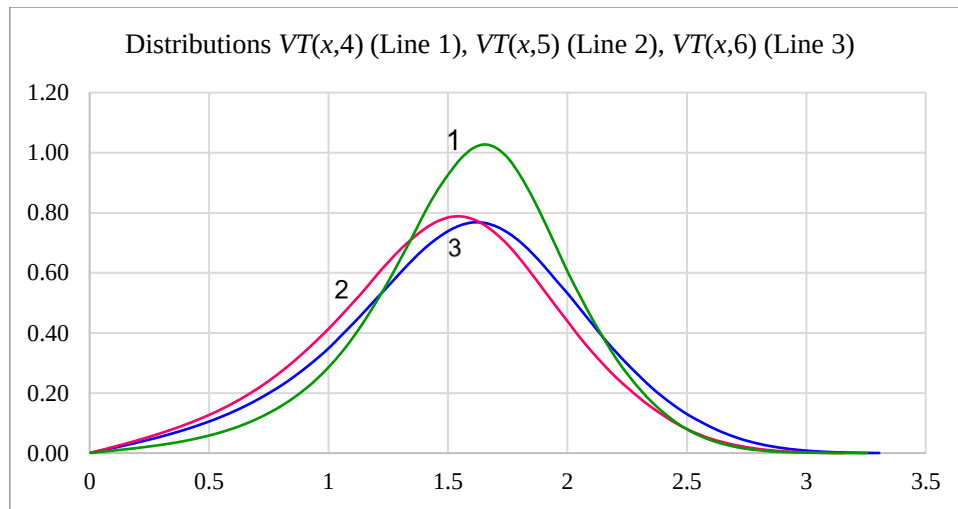


Figure 10: Log-asset distributions $VT(x, t)$ of levered firm near the debt maturity, $t = 4$ (Line 1), 5 (debt maturity, Line 2), 6 (Line 3).

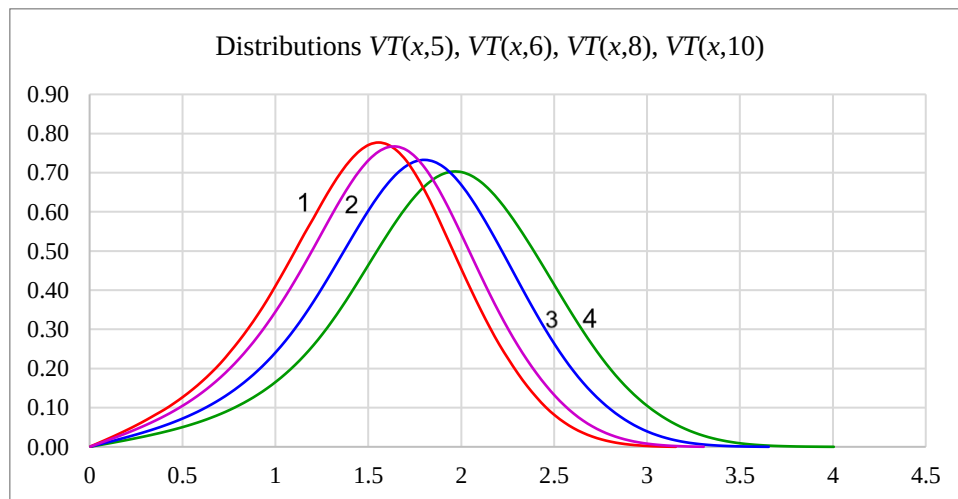


Figure 11: Log-asset distributions $VT(x, t)$ of levered firm in its recovery: $t = 5$ (debt maturity, Line 1), 6 (Line 2), 8 (Line 3) and 10 (Line 4).

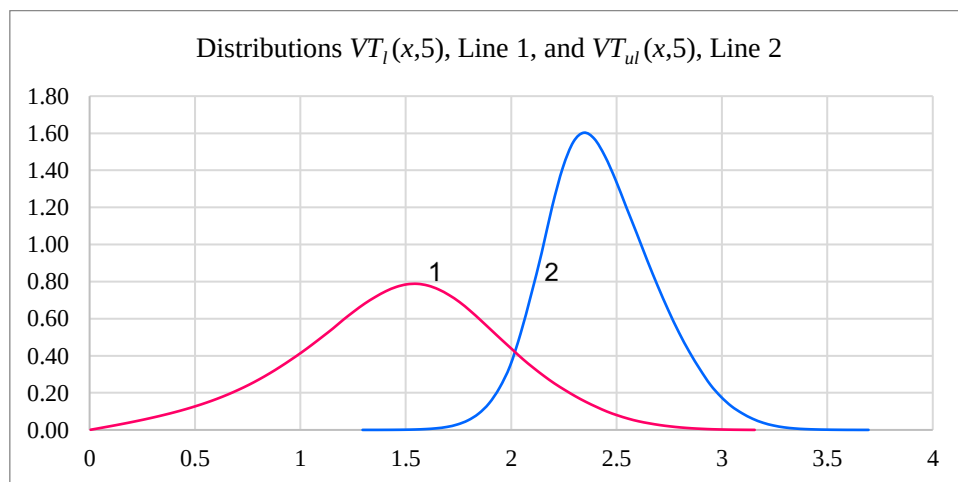


Figure 12: Log-asset distributions $VT_l(x, t)$ of levered firm at $t = 5$ (debt maturity, Line 1) and $VT_{ul}(x, t)$ of unlevered firm, $t = 5$ (Line 2).

Table 2: Evolution of Statistical Moments for the Asset Distribution of the Levered Firm After Debt and Tax Payments

Year	Mean return $H(t) - H_0$	Variance $VAR(t) - VAR_0$	Skewness	Kurtosis	$DPInt(t)$
0	0	0	0	0	0
1	-0.077	0.018	-3.65e-03	0.278	0
2	-0.164	0.047	-0.144	0.421	3.58e-07
3	-0.276	0.092	-0.338	0.903	3.67e-04
4	-0.425	0.158	-0.880	3.887	0.0101
5	-0.632	0.212	-1.462	6.461	0.0626
6	-0.539	0.228	-1.389	6.163	0.0536
7	-0.447	0.246	-1.318	5.883	0.0460
8	-0.356	0.264	-1.248	5.616	0.0395
9	-0.264	0.281	-1.180	5.364	0.0340
10	-0.172	0.292	-1.119	5.130	0.0294

After payment of the first portion of debt (Figure 6), the $VD(x,1)$ -distribution falls to the left not only of the $VB(x,1)$ -distribution but also of the initial distribution $V(x,0)$, indicating that debt payment of this size is detrimental to the asset growth. Figures 1 and 5, 6 demonstrate that the theorem of irrelevance (MMP1 1958) is wrong because it does not take into account of the firm's payments. Tax payment (Figure 7) makes the distribution $VT(x,1)$ right-skewed compared to the $VD(x,1)$ -distribution, but it has no significant effect on its location on the x -axis. Figure 8 shows a joint effect of earnings and debt and tax payments on the asset distribution; one can see that this effect is negative: The resulting $VT(x,1)$ -distribution falls left of the initial $V(x,0)$ -distribution. The same behavior of the $VT(x, t_k)$ -distribution in respect to the $VT(x, t_{k-1})$ -distribution can be seen for all years of debt payments, $k = 2, 3, 4, 5$ (Figures 9 and 10). Figures 8 and 9 reject the MMP3 (1963) stating a positive effect of debt on the firm value. The asset growth begins after the debt maturity when the debt is paid completely out and the firm reduces its payments to the business expenses and taxes only. Figure 12 contrasts the asset distributions of the levered and unlevered firms at the year of debt maturity corresponding to the lowest point in the asset development of the levered firm ($t = 5$). This picture exposes the myth about the beneficial effect of debt on the firm's development. Table 2 also contributes to this exposure: The difference between mean returns at $t = 5$, $H_U(5) - H_L(5) = 1.052$ is in favor of the unlevered firm; the variance difference, $VAR_L(5) - VAR_U(5) = 0.163$, is again in favor of the unlevered firm; the intensity of default probability is negligibly small for the unlevered firm and rather high for the levered firm, $DPInt(5) = 0.0626$. Table 2 shows that the losses in mean return $H(t) - H_0$, negative skewness, kurtosis, and the intensity of default probability grow fast in the first five years. They begin to decrease after the maturity when debt is completely paid off and the firm begins recovering. However, the recovery rates of all parameters are smaller than the rates of their degradation in the first five years. Table 2 and Figure 11 show that the firm needs five-plus years after the maturity for the partial recovery from the damages inflicted by debt.

Managers of the levered firm can consider its development as quite normal. They watch stochastic outcomes of asset returns, which are mostly positive. They compute mean returns and variances and explain the high variance values with a turbulent market. They

have no method to evaluate the intensity of default probability. Its estimates, provided by the mainstream financial experts using GBM equation for their forecasts, undervalue the default probability, inspiring false feelings of safety in the managers. Having no opportunity to compare various financing scenarios, managers usually choose between the trade-off and pecking-order theories recommended by the mainstream experts. Remember that about half of UK firms follow the false trade-off theory with target debt spreading from 0% to 80% (Beattie et al. 2006). High debt levels significantly deteriorate the firm's development, and about 60% of UK managers look for the alternative in pecking-order financing, which also pushes them to capital borrowing, though of a lesser scale. We show managers a real instrument for computing long-term consequences of debt financing.

CONCLUSION

Analyzing seminal papers of Modigliani and Miller (1958, 1963), Leland (1994), and Leland & Toft (1996), constituting the theoretical underpinning of the trade-off model of capital structure, we have shown that all of them are wrong. Following the ideas of the MMP3 (1963) and Kraus and Litzenberger (1973) that debt is beneficial for the firm, Leland (1994) has developed mathematical models making a quantitative basis for choosing the optimal capital structure. We have proved that the optimal capital structure promised by Leland is just the phantom caused by errors. His principal error stems from the approach that, taking into account tax benefits of debt, Leland disregards the increase in the firm's payments for debt redemption. Merton's GBM equation used by Leland admits no payments, unlike real business practice.

Using the Extended Merton model, which takes into account both inflows to and outflows from the firm's capital, we have shown that the cumulative effect of debt and tax negatively affects the firm value, its returns and stability, supporting the empirical study by Sarkar and Zapatero (2003), who have empirically shown that the higher the debt, the lesser the mean returns. All papers teaching how to choose the optimal debt level in static or dynamic conditions (Leland 1994, Leland & Toft 1996, Mella-Barral & Perraudin 1997, Goldstein et al. 2001, Hackbarth et al. 2006, Strebulaev 2007, Diamond & He 2014, etc.) are false and can inflict damages on the firm if it takes their recommendations for practical guidance. The same can be said about the agency theory suggesting a moderate debt in the firm's capital to mitigate the conflict of interests between the company's shareholders and managers (Grossman & Hart 1982, Jensen 1986). However, empirical studies by Beatty, Goodacre, and Thompson (2006) and Graham and Harvey (2001) show that the trade-off theory is very popular: 50% of British firms follow its recommendations exposing themselves to extra damages and risks.

Our conclusions on the effects of debt and tax on the firm value are as follows:

- (1) The theorem of irrelevance (MMP1 1958) is wrong, and debt does affect the firm value. The cause of this error is not the perfect market conditions, but the assumption of constant returns making real firms equivalent to the self-financing portfolio admitting no investments and payments.
- (2) Unlike the MMP3 (1963), the debt influence is negative, decreasing the levered firm value and its chances to survive in comparison to the identical unlevered firm in the same environment.

- (3) All studies, considering effects of debt and tax in the static or dynamic modes and based on the GBM equation, are wrong because they do not take account of investments and payments.
- (4) There is no optimal debt level in the spirit of Leland; debt negatively affects the levered firm value and stability because the tax shield is always accompanied by the rise in the firm's payments, offsetting the tax shield benefits.
- (5) The main factors depressing the levered firm are the size of debt payments added to the payments of this firm before debt and the length of debt maturity.
- (6) The debt pressure increases as the debt grows, keeping its maturity fixed; the greater the debt, the higher the probability of default. The effect of debt maturity is more complicated: For small maturities, the growth of maturity positively affects the firm value and survival, decreasing current debt payments; but starting with some optimal maturity, its growth deteriorates the levered firm value and its chances for survival. For each firm in its specific environmental conditions and for each reasonable debt, there is an optimal maturity minimizing negative effects of debt.
- (7) Corporate taxes cause the development of positive skewness in the firm's asset distribution, but do not significantly affect its location on the asset axis.
- (8) Selecting debt financing, managers should quantitatively analyze all possible consequences of this step. Our analysis of the cumulative effect of debt and tax on the firm value provides for an effective quantitative instrument, computing consequences of a capital structure decision. It can be helpful for practicing managers.

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