Archives of Business Research – Vol. 11, No. 9
Publication Date: September 25, 2023
DOI:10.14738/abr.119.15352.
Seretna-Sałamaj, D., & Szpara, A. (2023). Dynamic Management of Capital Levels in a Company as a Multi-Stage Decision-Making Process. Archives of Business Research, 11(9). 65-75.

Dynamic Management of Capital Levels in a Company as a Multi-Stage Decision-Making Process

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ABSTRACT

An investor makes investment decisions under conditions of uncertainty, i.e., when the effect of the investment cannot be predicted in advance. In order to achieve the desired outcome of the action, the decision-maker has to make a series of consecutive decisions. The decision-maker is guided by his or her own investment objective, and at each stage decides on an investment strategy. Depending on the strategy adopted, the investment returns can vary. Decisions related to giving the capital structure a target shape require a prior careful analysis of many factors, which are characterised by variability both in time and space. For this reason, it can be assumed that the capital structure is not fixed, as it changes according to the strength and direction of these factors. This article proposes a method for managing a company's capital level in a multi-stage decision-making process. For this purpose, the so-called recursive preferences of the decision-maker were used. A multi-stage model of capital management or the management of a given asset by an investor was considered. It was assumed that the dynamics of the capital level process from period to period is described by a certain recursive equation, which can be deterministic or stochastic.

Keywords: capital, investing, optimisation, decision-making, financial structure.

INTRODUCTION

The dynamics of change in the global economy means that effective business management requires continuous monitoring and decision-making in managing capital levels. The organisation's environment is becoming increasingly dynamic and unpredictable, the product life cycle is getting shorter and shorter, requiring organisations to constantly monitor and adapt to their environment.

The constant changes in today's markets (and especially financial markets) mean that the capital management process should be analysed from a dynamic perspective. Decisions related to giving the capital structure a target shape require a prior in-depth analysis of many factors, which are characterised by volatility both in time and in space. For this reason, it can be assumed that the capital structure is not fixed, as it changes according to the strength and direction of these factors.

One of the most essential skills of a financial manager is the ability to forecast future events and to plan in advance the financial steps that ensure the prevention of negative effects of phenomena that may occur in the future in connection with capital management.

Many models for managing the capital structure of a company are considered in the literature, but only in theoretical terms. Capital management models are not used in practice due to the poor knowledge of capital structure concepts and their importance in business management. Starting with the theory of F.M. Modigliani and M.H. Miller (1958), the issue of "enterprise value and the structure of finance" is one of the most important problems of contemporary finance theory, with rich references both in theory and in the practice of research on the structure of finance.

Decisions to manage the structure of fixed assets and their sources of financing are long-term decisions. "They are strategic in nature, and consequently their time horizon is much longer than current decisions. Within this group of decisions, one can speak of two types. Decisions shaping the structure of assets are referred to as investment decisions and decisions shaping the structure of financing sources are called financing decisions. Investment decisions are concerned with investment policy and determine in which assets the company should invest, while financing decisions specify how these investments should be financed1".

It seems that a particular point of interest in capital management is the concern to preserve liquidity. Operational management problems are predominant and therefore capital strategies have not yet acquired adequate importance. It becomes necessary to analyse the formation and selection of an appropriate capital structure, as well as to identify the factors influencing the structure of finances.

An investor makes investment decisions under conditions of uncertainty, i.e., when the effect of the investment cannot be predicted in advance. In order to achieve´ the desired outcome of the action´, the decision-maker has to make´ a series of subsequent decisions. Then the so-called multi-stage decision-making process is considered. The decision-maker is guided by his or her own investment objective and at each stage decides on an investment strategy. Depending on the strategy adopted, the returns on investment can vary. Both econometric models and dynamic programming methods are used in decision-making.

DYNAMIC ECONOMETRIC MODELS OF CORPORATE CAPITAL STRUCTURE MANAGEMENT

Understanding the mechanism of corporate capital structure management and the factors influencing it provides many important clues when building econometric models of target structure formation. Econometric methods also lead to the identification and quantification of causal and symptomatic relationships that are overlooked in theoretical models. A prerequisite for the construction of empirical models, the use of which would be effective in analysing the structure of a company's sources of finance, is the identification of the basic factors influencing the structure of finance, the identification of the cause-effect relationships occurring between them. It is then possible to identify the most important variables that define the state of the financial market, and then build econometric models based on the quantification of parameters on the basis of past statistical data. Econometric models quantifying cause-and-effect

relationships are constructed under conditions of uncertainty, as it is usually not possible to establish a complete set of causes defining the state of the modelled variable.

When talking about dynamic approaches to capital structure management using econometric models in the world literature, we can find many researchers including Jalivand, Fischer, Heinkel, Zechner, Ju, Leland, Benerjee, Hshmati, Wihlborg. Each of the proposed econometric models has a similar model building procedure in order to achieve the intended purpose and obtain the needed information. Knowing the cause-and-effect relationships describing the management of short-term as well as long-term capital structure, an econometric model can be created according to the procedure outlined in Figure 1.

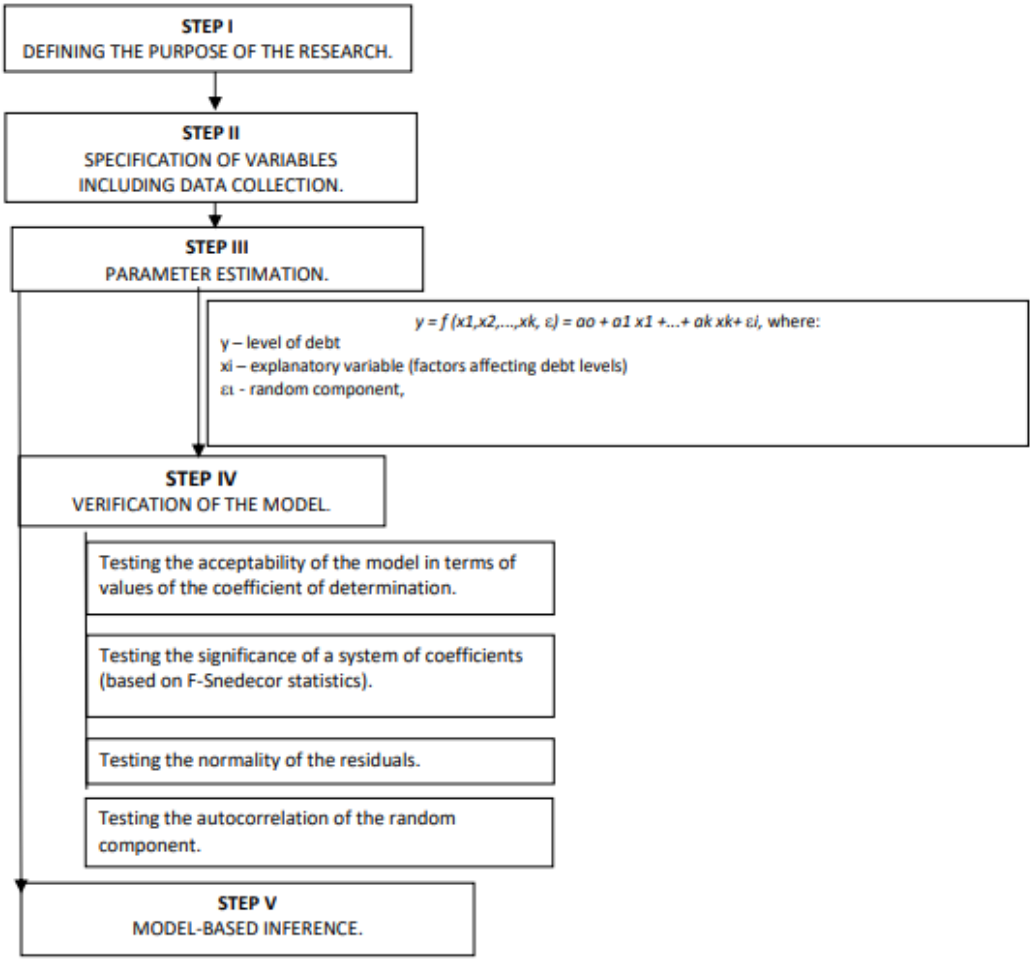


Fig: Econometric model in capital structure management
Source: own elaboration

THE MULTI-STAGE CAPITAL MANAGEMENT PROCESS - DYNAMIC PROGRAMMING

An investor makes investment decisions under conditions of uncertainty, i.e., when the effect of the investment cannot be predicted in advance. In order to achieve the desired outcome of the action, the decision-maker has to make a series of consecutive decisions. This is when the so-called multi-stage decision-making process is considered. The decision-maker is guided by his or her own investment objective, and at each stage decides on an investment strategy. Depending on the strategy adopted, the investment returns can vary. When an end result is

URL: http://dx.doi.org/10.14738/abr.119.15352

given and the objective is to determine a sequence of decisions leading to the desired end result, dynamic programming methods2 are used.

Dynamic programming is a technique mostly used to solve optimisation problems. The inventor of the technique is the American mathematician Richard Bellman, who was awarded the IEEE medal (medal of honour) for this discovery in 1979. The key to designing an algorithm using this technique lies in finding a recursive equation describing the optimal value of the objective function for a given problem as a function of the optimal values of the objective functions for sub-problems of smaller size. Divide the given problems into sub-problems (stages) and then solve them sequentially until an optimal solution is found. Dynamic programming, by solving sub problems, finds the optimal value of the objective function for the whole problem.

The solution to the last considered sub-problem is generally the value of the solution to the given problem. The solution of the considered problem is to find the optimal strategy, i.e., such a sequence of decisions for which the objective function reaches an extreme (depending on its practical sense - minimum or maximum).

In order for the dynamic programming method to be applied, the problem must have a Markov property, i.e., the optimal decisions at a certain stage of the process, do not depend on previous decisions, only on the state resulting from them.

From the Markov property follows Bellman's optimality principle, on which the dynamic programming method was based., ''An optimal strategy has the property that whatever the initial state was and whatever the initial decisions were, the remaining decisions must form an optimal strategy because of the state resulting from those initial decisions.''3

Dynamic programming uses Bellman's optimality principle independently of the algorithm. According to this principle, the optimal solution of dynamic programming problems has the property that the optimal solution for the k^{th} stage is at the same time the optimal solution for stages k+1, k+2, ..., N. Thus, the optimal solution for stage one is the optimal solution for the whole problem.

Due to Bellman's optimality principle, a dynamic programming problem is solved by starting with the search for the solution for the final N stage and then going backwards to find the solution for the N-1 stage.

The solution for stages N-1 and N obtained in this way is optimal regardless of how stage N-1 was reached.

By repeating stage after stage in the above manner, one arrives at the optimum solution for the first stage, and thus for the problem as a whole.

However, it is often not possible to explicitly describe the dynamics of the process and to estimate the consequences of the decisions made. In many cases, algorithm solvers try to obtain partial knowledge to estimate the probability distributions that describe the behaviour of the process depending on the decisions made and the values of the stage criteria associated with

these changes. This approach is called the stochastic approach. When evaluating the strategies under consideration, the concept of the expected value of the strategy, related to the probability distribution known to the decision-maker in the set of initial states, is used. To determine the optimal strategy, Bellman's optimality principle is used, this time in a stochastic version4.

The first to apply Bellman's equations to economics were Martin Beckmann and Richard Muth in their 1954 paper On the Solution to the Fundamental Equation of Inventory Theory. Martin Beckmann is also the author of a 1959 paper on consumption theory in which he used the Bellman equation. Another well-known application of the Bellman equation in economics is Robert Merton's 1973 work on the intertemporal asset pricing model (An Intertemporal Capital Asset Pricing Model). The solution to Merton's theoretical model, in which an investor makes a choice between income today or income in the future or capital gains, is a form of the Bellman equation. Nancy Stokey, Robert E. Lucas and Edward Prescott described stochastic and non-stochastic dynamic programming in detail and gave theorems for the existence of solutions to problems satisfying certain conditions. They also described many examples of modelling theoretical problems in economics using recursive methods (Recursive Methods in Economic Dynamics). This book presents applications of dynamic programming to the solution of a wide range of theoretical economic problems, including optimal economic growth, resource extraction, principal problems, public finance, business investment, asset pricing, factor supply and industrial organisation. Lars Ljungqvist and Thomas Sargent apply dynamic programming to study various theoretical problems in monetary policy, fiscal policy, taxation, economic growth, search theory and labour economics (Recursive Macroeconomic Theory). In contrast, Avinash Dixit and Robert Pindyck have applied dynamic programming to the evaluation of investment projects (Investment Under Uncertainty).

CAPITAL MANAGEMENT MODEL

This paper proposes a method for examining the level of a company's resources or natural resources, described by a certain recursive stochastic or deterministic equation. For this purpose, the so-called recursive preference of the decision maker was used.

Each project, capital stock requires decisions to be made at specific points in time and the selection of actions that would maximise the defined objective function, according to given conditions.

Recursive preferences can be very helpful in this issue, mainly for three reasons:

- they provide an opportunity and method to capture different investor preferences,
- they allow to distinguish between the decision-maker's attitude towards risk and the decision
- maker's attitude towards intertemporal substitution,
- allow the use of dynamic programming methods, such as backward induction in a model with a finite time horizon.

Recursive preferences, also known as recursive utility, have been codified by Koopmans Model5 in 1960 for the level of capital described by a deterministic equation. Kreps and Porteus6 They extended Koopmans' approach to stochastic models, and Epstein and Hynes7 as

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well as Epstein and Zin8 have incorporated the concept of intertemporal substitution into these preferences. Recursive utility has found application in natural resource management 9.

Recursive utility models represent an agent's preferences as the solution of non-linear functional equations. Therefore, as noted in Miao J. (2014), Economic Dynamics in Discrete Time, Cambridge: The MIT Press, Cambridge10 models can be analysed using dynamic programming methods that provide appropriate tools to capture the risk aversion of the decision maker while maintaining the same elasticity of intertemporal substitution.

A multi-stage consumption model with discrete time t= 1, 2, 3, ... is considered.

Let $c = (c_1, c_2, \ldots)$ where $c_t \geq 0$ for each t= 1,2, 3... be the consumption sequence of a given capital. A function U is called a recursive utility function if it satisfies the following equation:

$$U(c) = W\,(c_1, U\,(c_2, c_3, \ldots)),$$

where W is the so-called aggregating function, also known as an aggregate11. The above property of recurrence was introduced by Koopmans 12 in 1910. Later, many prominent economists considered different classes of utility functions U depending on the form of the aggregate W The following aggregate function is used in this article:

$$W\,(z_1, z_2) = ((1 - b)\, z_1^{\rho} + b z_2^{\rho})^{\frac{1}{\rho}},$$

where: $b \in (0,1)$ is the discount factor and ρ is a constant belonging to the interval (0,1].

This aggregate was introduced by Epstein and Hynes. If $\rho = 1$ is $W\,(z_1, z_2) = (1 - b)\, z_1 + b z_2$ and such a function leads to a standard discounted utility over a finite or infinite time horizon. It was introduced and analysed by Samuelson in 1937. However, as noted, such an aggregate applied to a given decision-making process (determining consumption and investment levels) can lead to extreme behaviour on the part of the decision-maker: consumption of all available capital or to zero consumption at each step.

The factor b denotes the subjective discount factor and usually $b = 1/1+d$, where d is the subjective discount rate). The factor ρ is used to determine the so-called elasticity of intertemporal substitution EIS.

OPTIMISATION PROCESS

The following multi-stage capital management process over a finite time horizon was considered

T ($T < +\infty$) with the following components:

- management period $t = 1, 2, \ldots, T$
- Xt - capital level during the period t; $X_1 = x$ - given level of capital at the beginning of the decision-making period,
- $C_t\,(\cdot)$ - period consumption function t,

- $I_t\,(X_t) = X_t - C_t(X_t)$ - investment function during the period t,
- b - subjective discount factor, $b = 1/1+d$ where d is a subjective rate,
- capital stock in the period $t + 1$ is defined by the recursive equation

$$X_{t+1} = R_{t+1}(X_t - C_t(X_t)),$$

where: $R_{t+1} = 1 + r_{t+1}$ i r_{t+1} means the rate of return on investment over the period $[t, t + 1)$.

The above model is taken from 13.

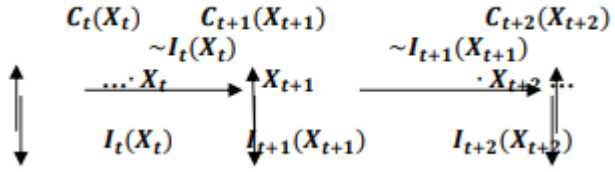


Fig.2: multi-stage decision-making process.
Source: own elaboration

The investor's objective is to determine the stream of consumption levels (C_1 (·), C_2 (·), C_3 (·), …, C_T (·)) and thus investments (I_1 (·), I_2 (·), I_3 (·), …, I_T (·)) so as to maximise the specified objective function. Since the process is multistage, it is assumed that the investor will use a power utility function to evaluate the future total utility, i.e., the guaranteed Kreps-Porteus equivalent. The investor's risk factor is then $\gamma - 1/x$ and therefore depends on the level of the good (capital)x. Through J_{t+1} denotes the total utility of the decision−maker calculated from the period $t + 1$ resulting from the optimal sequence of consumption ($C_{t+1} * (\cdot)$, $C_{t+2} * (\cdot)$, $C_{t+3} * (\cdot)$, …, $C_T * (\cdot)$) or equivalently the optimal sequence of investments ($I_{t+1} * (\cdot)$, $I_{t+2} * (\cdot)$, $I_{t+3} * (\cdot)$, …, I_{T^*} (·)). Through $C_l * (\cdot)$ optimum consumption during the period l. The decision-maker's optimisation problem is defined as follows: Problem (P)

$$\max_{0 \le c_t \le x_t} \left[(1-b)c_t^{\rho} + b\left(E\left[J_{t+1}^{\gamma}(X_{t+1}) | X_t \right]^{\frac{\rho}{\gamma}} \right]^{\frac{1}{\rho}}$$

on condition:

$$X_{t+1} = R_{t+1}\,(X_t - c_t) \text{ for each } t = 1, 2, \ldots, T.$$

Symbol E denotes the expected value operator calculated against the distribution of investment returns. Denoted by

$$J_{t+1}^{\gamma}(X_{t+1}) = \left(J_{t+1}(X_{t+1})\right)^{\gamma}$$

If the rate of return-on-investment r_{t+1} is not random, then:

$$\left(E\left[J_{t+1}^{\gamma}(X_{t+1}) | X_t\right]\right)^{\frac{1}{\gamma}} = J_{t+1}(X_{t+1})$$

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It follows from the principle of dynamic programming [117] that functions $J_t(\cdot), t = 1, 2, \dots, T$ satisfy the so–called Bellman equations, which means:

$$J_t(X_t) = \max_{0 \le c_t \le x_t} \left[(1-b)c_t^{\rho} + b\left(E\left[J_{t+1}^{\gamma}(X_{t+1}) | X_t \right]^{\frac{\rho}{\gamma}} \right]^{\frac{1}{\rho}}$$

Backward induction and dynamic programming principles were used to solve Problem (P) above. Figure 2 shows the solution to such a decision problem for a 3-period model. The very name, "backward induction", suggests that the solution should start from the last state of the X_3. Since there are three periods, at the final stage the decision–maker consumes all possible capital, i.e., X_3

So

$$C_3 * (X_3) = X_3 \text{ and } J_3(X_3) = X_3.$$

Thus, optimal investment $I_{3^*}(X_3) = 0$ (it is not profitable to invest, because the period of activity ends). The green arrows in Figure 2 lead from the state of X_3 to its previous state X_2. Using the aggregation function

$$W(z_1 z_2) = \left((1-b)z_1^{\rho} + bz_2^{\rho} \right)^{\frac{1}{\rho}}$$

and solving Problem P, the following one-period optimisation problems were obtained.

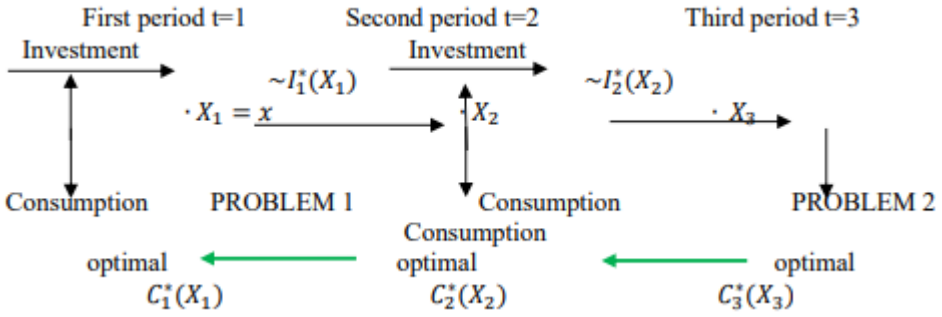


Fig.3: Backward induction.
Source: own elaboration

Problem 1 Problem (P) for $t = 2$.

$$\max_{0 \le c_2 \le x_2} \left[(1-b)c_2^{\rho} + b\left(E\left[X_3^{\gamma} | X_2 \right]^{\frac{\rho}{\gamma}} \right]^{\frac{1}{\rho}} \rightarrow J_2(X_2) \right.$$

on the condition that $X_3 = (X_2 - c_2) R_3$.
Solving Problem 1 for each level of capital X_2 optimum consumption was obtained $C_2 * (\cdot)$.

Hence, the optimal investment is $I_2 * (X_2) = X_2 - C_2 * (X_2)$. Then the final stage, the optimisation problem for period one, is left for consideration. Problem 2, Problem (P) for $t = 1$:

$$\max_{0 \le c_1 \le x_1} \left[(1-b)c_1^{\rho} + b \left(E\left[J_2^{\gamma}(X_2) | X_3 \right]^{\frac{\rho}{\gamma}} \right]^{\frac{1}{\rho}} \rightarrow J_1(X_1)$$

on the condition that $X_2 = (X_1 - c_1) R_2$. After solving the above problem, the optimal consumption function was obtained, i.e., the function $C_l * (\cdot)$. Then the optimal investment function is $I_l * (X_1) = X_1 - C_l * (X_l)$. By solving the problem, the investor receives the optimal function of the investment: $I_1 * (\cdot)$, $I_2 * (\cdot)$, $I_3 * (\cdot)$, and maximum usability $J_1 (x)$ in the three-period model if the initial capital is x.

Optimal consumption and investment strategies in a T-period model.

The next step is to calculate the optimal sequence of consumption and investment in the model with any time horizon T. It is assumed that optimal consumption is a linear function with respect to the level of capital, i.e., $C_t * (X_t) = a_t X_t$, then the Bellman equation takes the form:

$$J_t(X_t) = \max_{0 \le a_t \le 1} \left[(1-b)a_t^{\rho} X_t^{\rho} + b \left(E\left[J_{t+1}^{\gamma}(X_{t+1}) | X_t \right]^{\frac{\rho}{\gamma}} \right]^{\frac{1}{\rho}}$$

Assuming that the value function is also linear, i.e., $J_t (X_t) = a_t X_t$, $A_t \in [0, +\infty)$, obtained that:

$$E\left(J_{t+1}^{\gamma}(X_{t+1}) | X_t\right) = A_{t+1}^{\gamma} E\left[X_{t+1}^{\gamma} | X_t\right] = A_{t+1}^{\gamma} E\left[R_{t+1}^{\gamma}(X_t - a_t X_t)^{\gamma}\right] = A_{t+1}^{\gamma} E\left[R_{t+1}^{\gamma} \cdot X_t^{\gamma}\right](1-a_t)^{\gamma}$$
$$= A_{t+1}^{\gamma}(1-a_t)^{\gamma} X_t^{\gamma} E\left[R_{t+1}^{\gamma}\right]$$

The aim is to determine the value of A_t and a_t, which fulfil the Problem (P), i.e., the equation must hold:

$$A_t = \max_{0 \le a_t \le 1} \left[(1-b)a_t^{\rho} + (1-a_t)^{\rho} A_{t+1}^{\rho} b \left(E\left[R_{t+1}^{\gamma}\right]^{\frac{\rho}{\gamma}} \right]^{\frac{1}{\rho}}$$

The following numbers are solved:

$$a_t = \frac{1}{1 + \left[\frac{b}{1-b} A_{t+1}^{\rho} E\left[R_{t+1}^{\gamma}\right]^{\frac{\rho}{\gamma}} \right]^{\frac{1}{1-\rho}}}, a_T = 1$$

for $t = 1, 2, \ldots T$ and:

$$A_t = (1-b)^{\frac{1}{\rho}} a_t^{\frac{\rho-1}{\rho}}, A_{T+1} = 0$$

for $t = 1, 2, \ldots T$

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Thus:

$$C_t^*(x) = \frac{x}{1 + \left[ba_{t+1}^{\rho-1} E\left[R_{t+1}^{\gamma}\right]^{\frac{\rho}{\gamma}}\right]^{\frac{1}{1-\rho}}}$$

And:

$$I_t^*(x) = \frac{\left[ba_{t+1}^{\rho-1} E\left[R_{t+1}^{\gamma}\right]^{\frac{\rho}{\gamma}}\right]^{\frac{1}{1-\rho}}}{1 + \left[ba_{t+1}^{\rho-1} E\left[R_{t+1}^{\gamma}\right]^{\frac{\rho}{\gamma}}\right]^{\frac{1}{1-\rho}}} x$$

SUMMARY

Global changes in economic conditions - the increasing volatility of commodity prices, exchange rates, interest rates - amplify the risk of companies' operations, forcing the need to take into account the impact of risk on the effects of decisions on sources of finance. The volatility of business conditions creates the need to respond to changes in the environment - companies can adapt to changes in the environment by changing the scale of operations, postponing or abandoning certain investments when they prove unprofitable, etc. Recent literature presents the view that the structure of a company's finances should take into account the relationship between the company's financial management and the type and size of its activities and assets. Consequently, it is not possible to determine the optimum structure of finances for all companies; however, it is possible to determine an effective structure of finances for a selected company. It is also possible to develop general assumptions and tools for managing the structure of a company's finances. The theories for the formation of an optimal corporate finance structure are relatively little known compared to other aspects of corporate finance management and are rarely described in the national specialist literature. In most cases in business entities, long-term financial decisions are made in a subjective manner unsupported by detailed analyses of companies and the environment. The uncertainty surrounding business activities creates the need for a dynamic view of the structure of finances under changing conditions. The paper considers a decision-making model in which the decision-maker in each step has to decide how much to consume versus how much finance to invest. A method for managing a company's capital level in a multi-step decision-making process is proposed. For this purpose, the so-called recursive preferences of the decision-maker were used. A multi-stage model of capital management or the management of a given asset by an investor was considered. It was assumed that the dynamics of the capital level process from period to period is described by a certain recursive equation, which can be deterministic or stochastic. It was assumed that the investor aggregates the future total utility and the immediate utilities derived from the consumption of a given good in a given period using the aggregating function introduced by Epstein and Zin. The investor uses the guaranteed Kreps-Porteus equivalence to estimate future total utility. The corresponding formulas for optimal consumption and investment strategies were derived. Optimal consumption strategies were determined, taking into account the impact of the investor's risk factor.

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