**Archives of Business Research – Vol. 11, No. 6**
**Publication Date:** June 25, 2023
**DOI**:10.14738/abr.116.14871.
Berketis, N. G. (2023) A Chain-Ladder Analysis of P&I Claims - 2021. Archives of Business Research, 11(6). 43-59.

SERVICES FOR SCIENCE AND EDUCATION

# A Chain-Ladder Analysis of P&I Claims - 2021[i]

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**Keywords:** claim, CL method, run-off triangle, claim reserving, P. & I.

According to the latest Market Review for 2021/2022 prepared by Lockton. [2] the latest financial results from the International Group of P&I Clubs suggests that we are now in the full swing of a hardening market with premium rises due to be requested over thenext year or, perhaps, longer – whether this be due to individual Club performance or the renewal of the International Group General Excess of Loss programme, or both.

These anticipated rises come at a time when the shipping market has only just showed signsof improving and while operational expenses continue to rise – from the ongoing procedural difficulties continuing from the pandemic to the exclusion of historically peripheral risks such as cyber-attacks.

However, and despite these difficulties, the financial results of the P&I insurers are supporting the Club's argument that rises are required. Therein lies one of the unique traits of a mutual: both the financial prosperity and burdens of the Clubs are shared across its members[2].

The insurance industry, unlike other industries, does not sell products as such but promises. An insurance policy is a promise by the Insurer to the Policyholder to pay for future claims against an upfront received premium.

As a result, Insurers don't know the upfront cost for their service, but rely on historical data analysis and judgement to predict a sustainable price for their offering. In General Insurance (or Non-Life Insurance, e.g., motor, property, and casualty insurance) most Policies run for a period of 12 months. Nevertheless, the claims payment process can take years or even decades. Therefore, not even the delivery date of their product is often unknown to Insurers.

Losses arising from casualty insurance can take a long time to settle and, even when the claims are acknowledged, it may take time to establish the extent of the claims' settlement cost. Claims can take years to materialise. A complex and costly example is the claims from asbestos liabilities, particularly those in connection with mesothelioma and lung damage arising from prolonged exposure to asbestos. A research report by a working party of the Institute and Faculty of Actuaries estimated that the un-discounted cost of UK mesothelioma-related claims to the UK Insurance Market for the period 2009 to 2050 could be around £10bn. The cost for asbestos related claims in the US for the worldwide insurance industry was estimated to be around $120bn in 2002.

Thus, it should come as no surprise that the biggest item on the liabilities side of an Insurer's balance sheet is often the provision or reserves for future claims payments. Those reserves can be broken down in case reserves (or outstanding claims), which are losses already reported to the insurance company and losses that are incurred but not reported (IBNR) yet.

The analysis is based on most recent version of *R* (Version 4.1.2 – 01st November 2021), an integrated language and environment for statistical computing and graphics. *R* provides a wide variety of statistical and graphical techniques.

The estimated cost of notified pool claims (in USD 000,000) is as follows:

| YEAR | 12M | 24M | 36M | 48M | 60M | 72M | 84M | 96M | 108M | 120M |
|---|---|---|---|---|---|---|---|---|---|---|
| 2011/12 | 2310 | 2779 | 2808 | 2896 | 2893 | 2887 | 2844 | 2812 | 2802 | 2799 |
| 2012/13 | 3686 | 4539 | 4670 | 4651 | 4463 | 4186 | 4036 | 3927 | 3834 | NA |
| 2013/14 | 2798 | 3270 | 3640 | 3649 | 4116 | 4086 | 4231 | 4306 | NA | NA |
| 2014/15 | 1796 | 1936 | 2045 | 2158 | 2213 | 2129 | 2065 | NA | NA | NA |
| 2015/16 | 1984 | 2766 | 2840 | 2827 | 2914 | 2955 | NA | NA | NA | NA |
| 2016/17 | 840 | 1259 | 1450 | 1365 | 1402 | NA | NA | NA | NA | NA |
| 2017/18 | 2272 | 2696 | 2897 | 3105 | NA | NA | NA | NA | NA | NA |
| 2018/19 | 3061 | 4558 | 4900 | NA | NA | NA | NA | NA | NA | NA |
| 2019/20 | 2592 | 4417 | NA | NA | NA | NA | NA | NA | NA | NA |
| 2020/21 | 4629 | NA | NA | NA | NA | NA | NA | NA | NA | NA |

This triangle shows the known values of loss from each origin year and of annual evaluations thereafter. For example, the known values of loss originating from the 2013/14 exposure period are 2798, 3270, and 3640 as of year ends 2013, 2012, and 2011, respectively. The latest diagonal – i.e., the vector 2799, 3834, . . . 4629 from the upper right to the lower left – showsthe most recent evaluation available.

The column headings – 1, 2, . . ., 10 – hold the ages (in years) of the observations in the column relative to the beginning of the exposure period. For example, for the 2014/15 originyear, the age of the 2045 value, evaluated as of 20/02/2018, is three years.

The objective of a reserving exercise is to forecast the future claims development in the bottom right corner of the triangle and potential further developments beyond development age 10. Eventually, all claims for a given origin period will be settled, but it is not always obvious to judge how many years or even decades it will take.

We speak of long and short tail business depending on the time it takes to pay all claims.

In order proceed with our analysis, we first plotted the data to get an overview. Figure 1 that follows shows the claims development chart for the past 10 years.

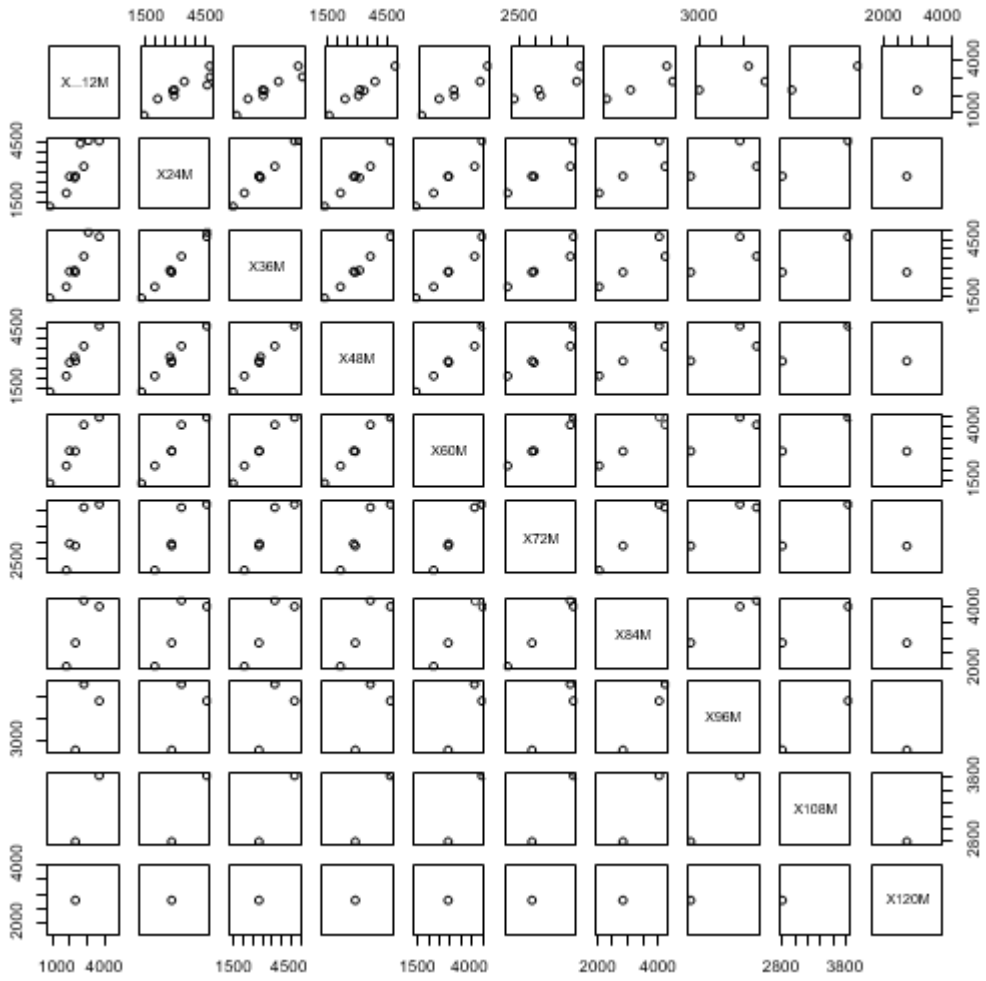



**Figure 1 – Claims Development Chart for the past 10 Years**

## CHAIN-LADDER METHODS

The classical chain-ladder is a deterministic algorithm to forecast claims based on historical data. It assumes that the proportional developments of claims from one development periodto the next are the same for all origin years.

### Basic Idea

Most commonly as a first step, the age-to-age link ratios are calculated as the volume weighted average development ratios of a cumulative loss development triangle from one development period to the next $C_{ik}$, $i$, $k = 1, \ldots, n$.

$$f_k = \frac{\sum_{i=1}^{n-k} C_{i,\ k+1}}{\sum_{i=1}^{n-k} C_{i,k}}$$

[1] 1.3224612 1.0607907 1.0147912 1.0259318 0.9785529 0.9915713 0.9940599 0.9847158 0.9989293.



**URL:** http://dx.doi.org/10.14738/abr.116.14871

Often it is not suitable to assume that the oldest origin year is fully developed. A typical approach is to extrapolate the development ratios, e.g., assuming a log-linear model.[1] 1.000105.

Figure 2 below shows the Log-linear extrapolation of age-to-age factors.

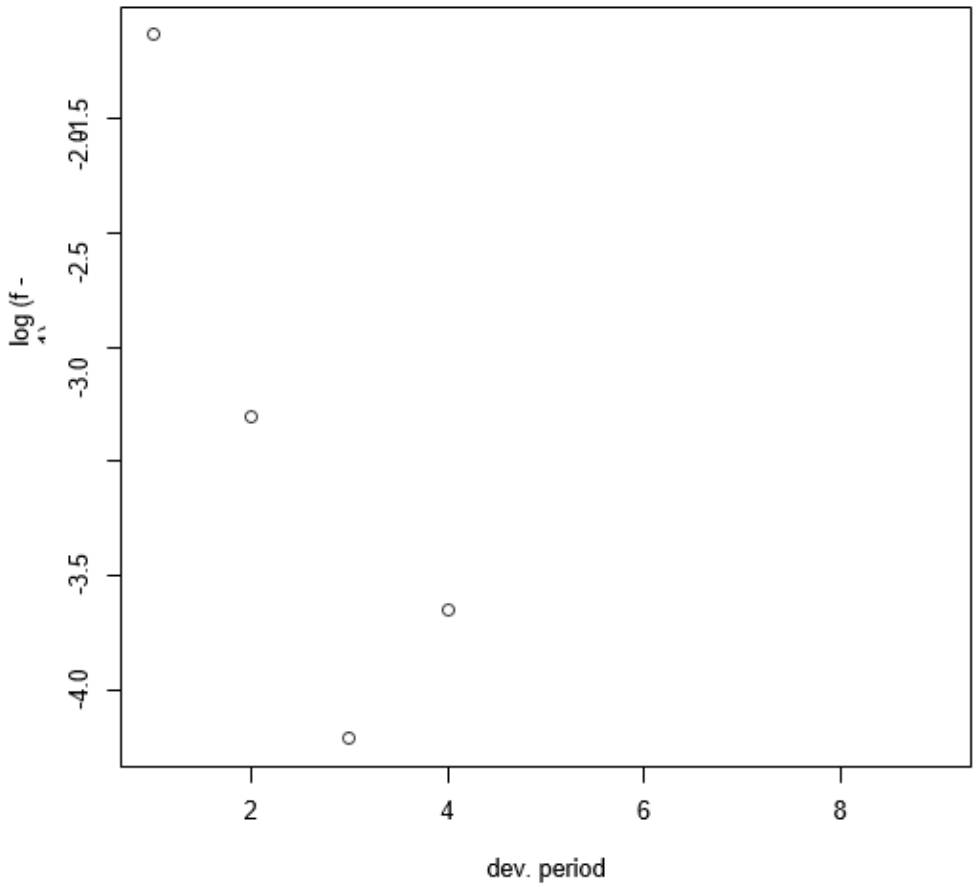



**Figure 2 Log-linear extrapolation of age-to-age factors**

The age-to-age factors allow us to plot the expected claims development patterns.

This is shown on Figure 3

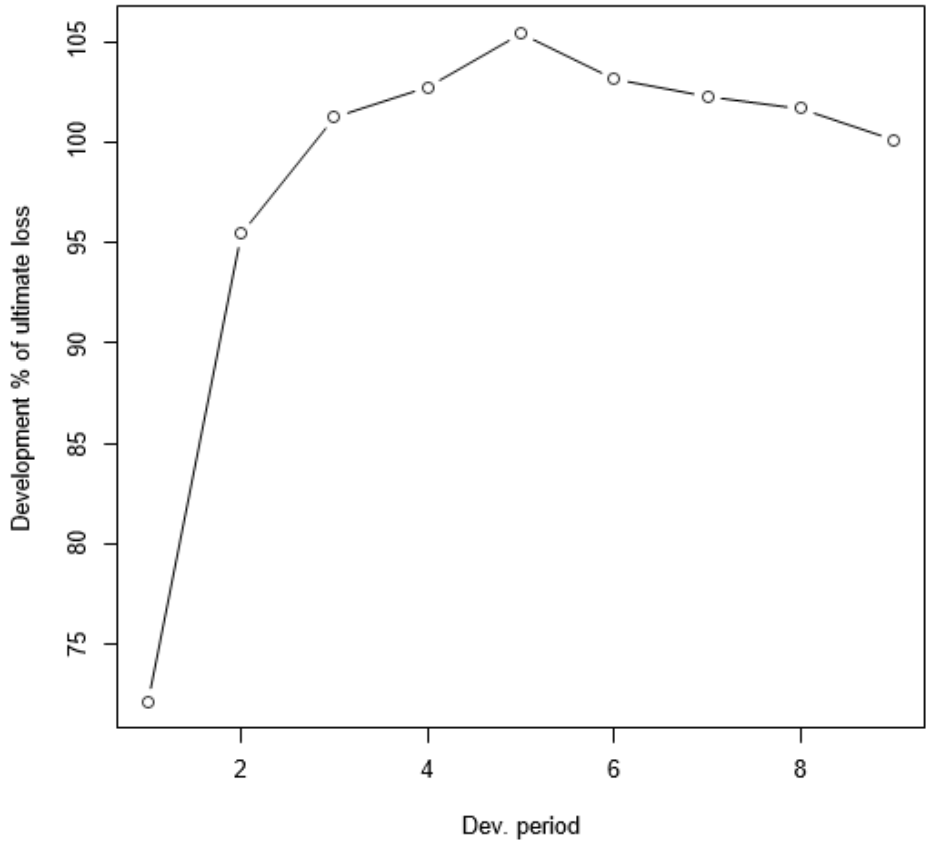



**Figure 3 Expected claims development pattern**

The link ratios are then applied to the latest known cumulative claims amount to forecast the next development period. The squaring of the triangle is calculated below, where an ultimate column is appended to the right to accommodate the expected development beyond the oldest age (10) of the triangle due to the tail factor (1.000105) being greater than unity.

**Table 1 – Squaring of the Triangle**

| | X12M | X24M | X36M | X48M | X60M | X72M | X84M | X96M | X108M | X120M | Ult |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2310 | 2779 | 2808 | 2896 | 2893 | 2887 | 2844 | 2812 | 2802 | 2799 | 2799 |
| 2 | 3686 | 4539 | 4670 | 4651 | 4463 | 4186 | 4036 | 3927 | 3834 | 3830 | 3830 |
| 3 | 2798 | 3270 | 3640 | 3649 | 4116 | 4086 | 4231 | 4306 | 4240 | 4236 | 4236 |
| 4 | 1796 | 1936 | 2045 | 2158 | 2213 | 2129 | 2065 | 2053 | 2021 | 2019 | 2019 |
| 5 | 1984 | 2766 | 2840 | 2827 | 2914 | 2955 | 2930 | 2913 | 2868 | 2865 | 2865 |
| 6 | 840 | 1259 | 1450 | 1365 | 1402 | 1372 | 1360 | 1352 | 1332 | 1330 | 1330 |
| 7 | 2272 | 2696 | 2897 | 3105 | 3186 | 3117 | 3091 | 3073 | 3026 | 3022 | 3023 |
| 8 | 3061 | 4558 | 4900 | 4972 | 5101 | 4992 | 4950 | 4921 | 4845 | 4840 | 4841 |
| 9 | 2592 | 4417 | 4686 | 4755 | 4878 | 4773 | 4733 | 4705 | 4633 | 4628 | 4629 |
| 10 | 4629 | 6122 | 6494 | 6590 | 6761 | 6616 | 6560 | 6521 | 6421 | 6414 | 6415 |

The total estimated outstanding loss under this method is about 36,000. It was calculated as 35,987.



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This approach is also called Loss Development Factor (LDF) method. More generally, the factors used to square the triangle need not always be drawn from the dollar weighted averages of the triangle. Other sources of factors from which the actuary may select link ratiosinclude simple averages from the triangle, averages weighted toward more recent observations or adjusted for outliers, and benchmark patterns based on related, morecredible loss experience.

Also, since the ultimate value of claims is simply the product of the most current diagonal and the cumulative product of the link ratios, the completion of interiorof the triangle is usually not displayed in favor of that multiplicative calculation.

## MACK CHAIN-LADDER

Thomas Mack published in 1993[ii] a method which estimates the standard errors of the chain-ladder forecast without assuming a distribution under three conditions.

Following the notation of Mack[iii] let $C_{ik}$ denote the cumulative loss amounts of origin period (e.g., accident year*)* $i$ = 1, . . ., m, with losses known for development period (e.g., development year) $k \leq n + 1 - i$.

To forecast the amounts $C_{ik}$ for $k > n+1-i$ the Mack chain-ladder-model assumes:

$$\text{CL1: E } [F_{ik} \text{I} C_{i1}, C_{i2}, \dots \dots \dots, C_{ik}] \; = \; f_k \text{with } F_{ik} \; = \frac{C_{i,k+1}}{C_{ik}} \qquad (2)$$

$$CL2: V_{ar}(\frac{C_{,i,k+1}}{C_{ik}} \text{I} C_{i1}, C_{i2}, \dots \dots, C_{ik}) = \frac{\sigma_k^2}{w_{ik} C_{ik}^{\alpha}} \qquad (3)$$

$$CL3: \{C_{i1}, \dots \dots \dots, C_{in}\}, \{C *_1, \dots \dots \dots, C *_n, \text{ are independent for origin period } i \neq \text{ j} \quad (4)$$

with $w_{ik} \in [0; 1]$, $\alpha \in \{0, 1, 2\}$.

If these assumptions hold, the Mack chain-ladder model gives an unbiased estimator for IBNR (Incurred but Not Reported) claims.

The Mack chain-ladder model can be regarded as a weighted linear regression through the origin for each development period: lm (y ~ x + 0, weights=w/x^(2- alpha)), where y is the vector of claims at development period k + 1 and x is the vector of claims at development period k.

The Mack method is implemented in the ChainLadder package via the function MackChainLadder.

We therefore applied the MackChainLadder function to our triangle:

**Table 2-Application of the Mack Chain-Ladder Method**

| | Latest | Dev. To. Date | Ultimate | IBNR | Mack. S.E. | CV (IBNR) |
|---|---|---|---|---|---|---|
| 1 | 2,799 | 1.000 | 2,799 | 0.0 | 0.0 | NaN |
| 2 | 3,834 | 1.001 | 3,830 | -4.1 | 43.4 | -10.580 |
| 3 | 4,306 | 1.017 | 4,236 | -70.4 | 83.0 | -1.180 |
| 4 | 2,065 | 1.023 | 2,019 | -45.8 | 86.7 | -1.893 |
| 5 | 2,955 | 1.031 | 2,865 | -89.9 | 159.8 | -1.778 |
| 6 | 1,402 | 1.054 | 1,330 | -71.8 | 123.8 | -1.723 |
| 7 | 3,105 | 1.027 | 3,022 | -82.6 | 281.5 | -3.407 |
| 8 | 4,900 | 1.012 | 4,840 | -59.9 | 405.2 | -6.770 |
| 9 | 4,417 | 0.954 | 4,628 | 211.3 | 425.9 | 2.016 |
| 10 | 4,629 | 0.722 | 6,414 | 1,785.5 | 935.3 | 0.524 |

| **Totals** | |
|---|---|
| Latest: | 34,412.00 |
| Dev: | 0.96 |
| Ultimate: | 35,984.28 |
| IBNR: | 1,572.28 |
| Mack. S.E.: | 1,420.95 |
| CV(IBNR): | 0.90 |

Executing Mack Chain Ladder will print the following columns of information per accident year (origin period):

1. **Latest**: the claim amount for the last development period
2. **Dev.To.Date**: the development to date or the ratio of the latest over the predicted ultimate
3. **Ultimate**: predicted ultimate claim
4. **IBNR**: the predicted IBNR reserve
5. **Mack.S.E.**: the standard error, or the standard deviation of the bounds for thepredicted ultimate and IBNR since the estimate is unbiased (shown in Mack's 1999
6. paper). In other words, since the S.E given is equal to one standard deviation, a confidence interval for the true ultimate value can be found using the standard errorand the predicted ultimate.
7. **CV(IBNR)**: coefficient of variation, or the ratio of the standard error over the predicted IBNR

The bottom output gives a total or sum of the latest, ultimates, IBNRs. It also gives the standard error of the total ultimate (this is not the total of the standard errors). The development to date factor is the ratio of the total latest against the total ultimate, and the CV(IBNR) is the percentage of the total standard error in the total IBNR.

If the CV (absolute value) is greater than 25%, then another model or a log linear regression should be used.


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We can access the loss development factors and the full triangle via:

[1] 1.3224612 1.0607907 1.0147912 1.0259318 0.9785529 0.9915713 0.9940599 0.9847158 0.9989293 [10] 1.0000000

**Table 3-Full Triangle following Application of Mack Chain-Ladder Method**

| origin | X12M | X24M | X36M | X48M | X60M | X72M | X84M | X96M | X108M | X120M |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2310 | 2799.000 | 2808.000 | 2896.000 | 2893.000 | 2887.000 | 2844.000 | 2812.000 | 2802.000 | 2799.000 |
| 2 | 3686 | 4539.000 | 4670.000 | 4651.000 | 4463.000 | 4186.000 | 4036.000 | 3927.000 | 3834.000 | 3829.895 |
| 3 | 2798 | 3270.000 | 3640.000 | 3649.000 | 4116.000 | 4086.000 | 4231.000 | 4306.000 | 4240.186 | 4235.647 |
| 4 | 1796 | 1936.000 | 2045.000 | 2158.000 | 2213.000 | 2129.000 | 2065.000 | 2052.734 | 2021.359 | 2019.195 |
| 5 | 1984 | 2766.000 | 2840.000 | 2827.000 | 2914.000 | 2955.000 | 2930.093 | 2912.688 | 2868.170 | 2865.100 |
| 6 | 840 | 1259.000 | 1450.000 | 1365.000 | 1402.000 | 1371.931 | 1360.368 | 1352.287 | 1331.618 | 1330.193 |
| 7 | 2272 | 2696.000 | 2897.000 | 3105.000 | 3185.518 | 3117.198 | 3090.925 | 3072.564 | 3025.603 | 3022.363 |
| 8 | 3061 | 4558.000 | 4900.000 | 4972.477 | 5101.422 | 4992.012 | 4949.936 | 4920.533 | 4845.326 | 4840.139 |
| 9 | 2592 | 4417.000 | 4685.512 | 4754.816 | 4878.118 | 4773.496 | 4733.262 | 4705.146 | 4633.232 | 4628.271 |
| 10 | 4629 | 6121.673 | 6493.814 | 6589.865 | 6760.752 | 6615.753 | 6559.992 | 6521.025 | 6421.356 | 6414.481 |

To plot that Mack's assumption are valid review the residual plots, we see no trends in eitherof them. Please refer to the Figure 4 that follows:

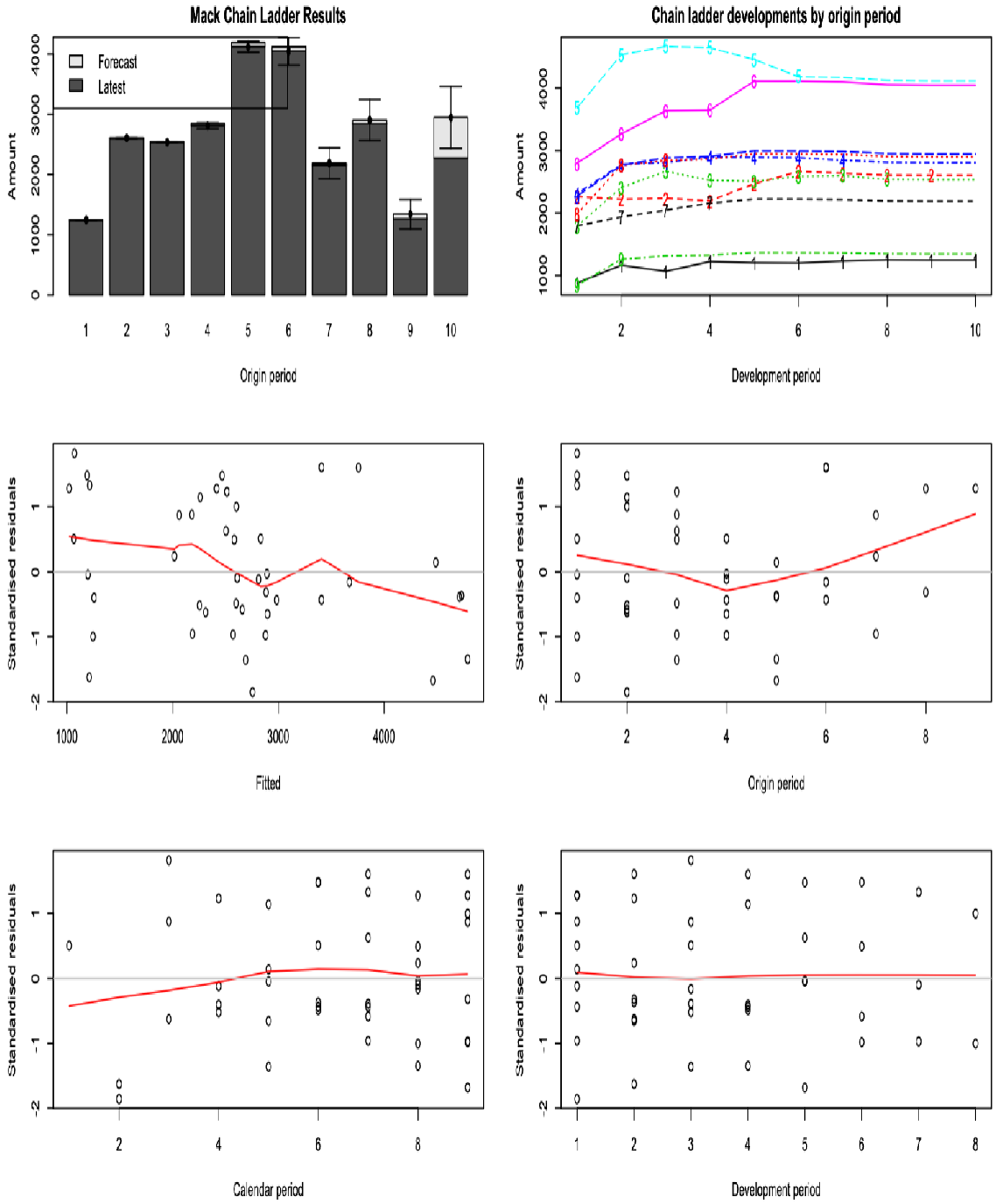



**Figure 4**

We can now plot the development, including the forecast and estimated standard errors by origin period by setting the argument lattice=TRUE. Figure 5 that follows depicts same:



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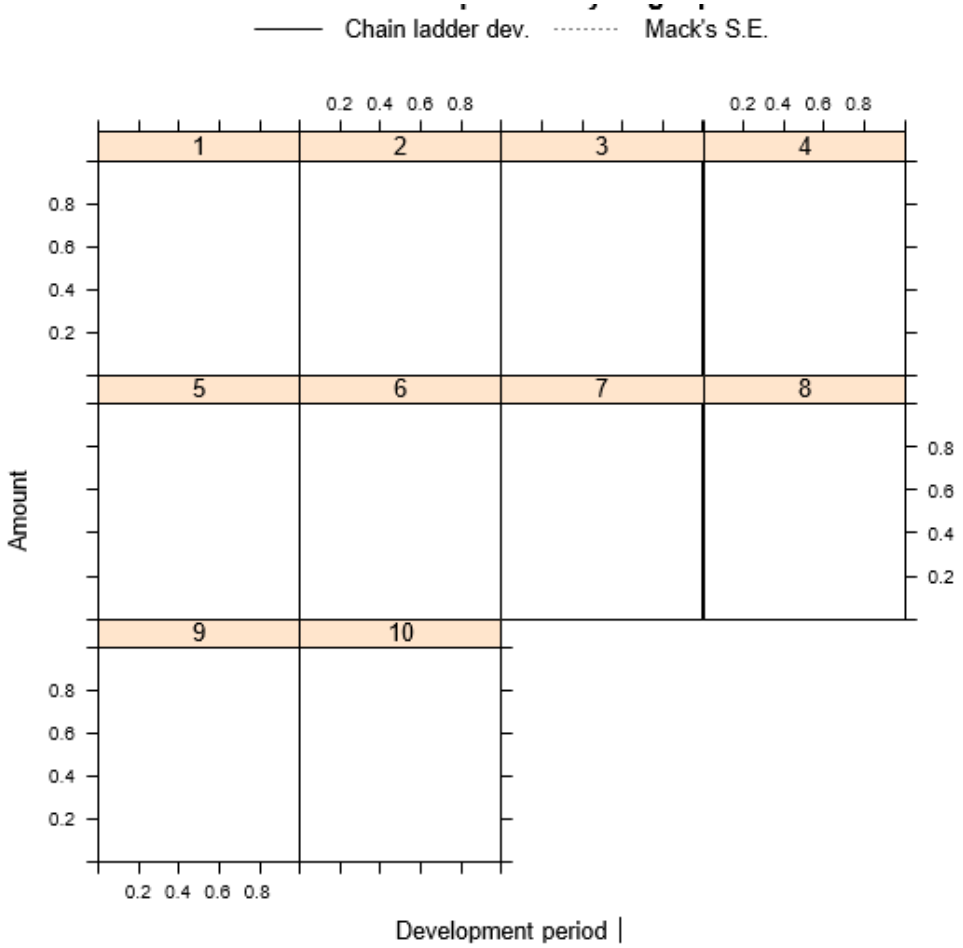



**Figure 5 Chain ladder developments by origin period**

The weights argument allows for the selection of a subset of the triangle for the projections. For example, to use only the last 5 calendar years of the triangle, set the weights as follows:

| | [,1] | [,2] | [,3] | [,4] | [,5] | [,6] | [,7] | [,8] | [,9] | [,10] |
|---|---|---|---|---|---|---|---|---|---|---|
| [1,] | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 |
| [2,] | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | NA |
| [3,] | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | NA | NA |
| [4,] | 0 | 0 | 1 | 1 | 1 | 1 | 1 | NA | NA | NA |
| [5,] | 0 | 1 | 1 | 1 | 1 | 1 | NA | NA | NA | NA |
| [6,] | 1 | 1 | 1 | 1 | 1 | NA | NA | NA | NA | NA |
| [7,] | 1 | 1 | 1 | 1 | NA | NA | NA | NA | NA | NA |
| [8,] | 1 | 1 | 1 | NA | NA | NA | NA | NA | NA | NA |
| [9,] | 1 | 1 | NA | NA | NA | NA | NA | NA | NA | NA |
| [10,] | 1 | NA | NA | NA | NA | NA | NA | NA | NA | NA |

MackChainLadder(Triangle = GGG123, weights = weights, est.sigma = "Mack")

| | Latest | Dev. To. Date | Ultimate | IBNR | Mack. S.E. | CV (IBNR) |
|---|---|---|---|---|---|---|
| 1 | 2,799 | 1.000 | 2,799 | 0.0 | 0.0 | NaN |
| 2 | 3,834 | 1.001 | 3,830 | -4.1 | 43.4 | -10.580 |
| 3 | 4,306 | 1.017 | 4,236 | -70.4 | 83.0 | -1.180 |
| 4 | 2,065 | 1.023 | 2,019 | -45.8 | 86.7 | -1.893 |
| 5 | 2,955 | 1.031 | 2,865 | -89.9 | 159.8 | -1.778 |
| 6 | 1,402 | 1.058 | 1,325 | -77.4 | 127.9 | -1.653 |
| 7 | 3,105 | 0.994 | 3,123 | 18.2 | 268.5 | 14.762 |
| 8 | 4,900 | 0.971 | 5,048 | 147.8 | 430.7 | 2.915 |
| 9 | 4,417 | 0.906 | 4,876 | 459.2 | 455.0 | 0.991 |
| 10 | 4,629 | 0.614 | 7,538 | 2,909.5 | 1,145.5 | 0.394 |

| **Totals** | |
|---|---|
| Latest: | 34,412.00 |
| Dev: | 0.91 |
| Ultimate: | 37,659.08 |
| IBNR: | 3,427.08 |
| Mack. S.E.: | 1,658.96 |
| CV(IBNR): | 0.51 |

## BOOTSTRAP CHAIN-LADDER

The BootChainLadder function uses a two-stage bootstrapping / simulation approach following the paper by England and Verrall[iv]. In the first stage an ordinary chain-ladder methodis applied to the cumulative claims' triangle. From this we calculate the scaled Pearson residuals which we bootstrap R times to forecast future incremental claims payments via thestandard chain-ladder method. In the second stage we simulate the process error with the bootstrap value as the mean and using the process distribution assumed. The set of reservesobtained in this way forms the predictive distribution, from which summary statistics such asmean, prediction error or quantiles can be derived.

Boot Chain Ladder (Triangle = GGG123, R = 999, process.distr = "gamma")

| | Latest Mean | Ultimate Mean | IBNR | IBNR S.E. | IBNR 75% | IBNR 95% |
|---|---|---|---|---|---|---|
| 1 | 2,799 | 2,799 | 0.0 | 0 | 0.00e+00 | 0.0 |
| 2 | 3,834 | 3,830 | -4.47 | 96.4 | 1.05e-03 | 86.2 |
| 3 | 4,306 | 4,232 | -73.99 | 236.3 | 3.10e-07 | 183.6 |
| 4 | 2,065 | 2,009 | -56.15 | 175.0 | 2.03e-28 | 105.9 |
| 5 | 2,955 | 2,869 | -86.26 | 222.8 | -3.92e-01 | 195.9 |
| 6 | 1,402 | 1,322 | -79.78 | 167.6 | -2.17e-01 | 39.8 |
| 7 | 3,105 | 3,033 | -71.68 | 330.8 | 4.68e+01 | 403.6 |
| 8 | 4,900 | 4,818 | -81.63 | 452.7 | 1.50e+02 | 624.0 |
| 9 | 4,417 | 4,629 | 211.96 | 546.3 | 4.82e+02 | 1,118.1 |
| 10 | 4,629 | 6,350 | 1,721.00 | 1,028.8 | 2.32e+03 | 3,622.0 |


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| **Totals** | |
|---|---|
| Latest: | 34,412 |
| Mean Ultimate: | 35,891 |
| Mean IBNR: | 1,479 |
| IBNR S.E. | 1,644 |
| Total IBNR 75%: | 2,469 |
| Total IBNR 95%: | 4,220 |

The BootChainLadder is a model that provides a predicted distribution for the IBNR values fora claims' triangle. However, this model predicts IBNR values by a different method than the previous model. First, the development factors are calculated and then they are used in a backwards recursion to predict values for the past loss triangle. Then the predicted values and the actual values are used to calculate Pearson residuals.

Using the adjusted residuals and the predicted losses from before, the model solves for the actual losses in the Pearson formula and forms a new loss triangle. The steps for predicting past losses and residuals are then repeated for this new triangle. After that, the model uses chain ladder ratios to predict the future losses then calculates the ultimate and IBNR values like in the previous Mack model. This cycle is performed R times, depending on the argumentvalues in the model (default is 999 times). The IBNR for each origin period is calculated from each triangle (the default 999) and used to form a predictive distribution, from whichsummary statistics are obtained such as mean, prediction error, and quantiles.

The output has some of the same values as the Munich and Mack models did. The Mean andSD IBNR is the average and the standard deviation of the predictive distribution of the IBNRsfor each origin year

The output also gives the 75% and 95% quantiles of the predictive distribution of IBNRs, in other words 95% or 75% of the predicted IBNRs lie at or below the given values.

The above also appear on following Figure 6:

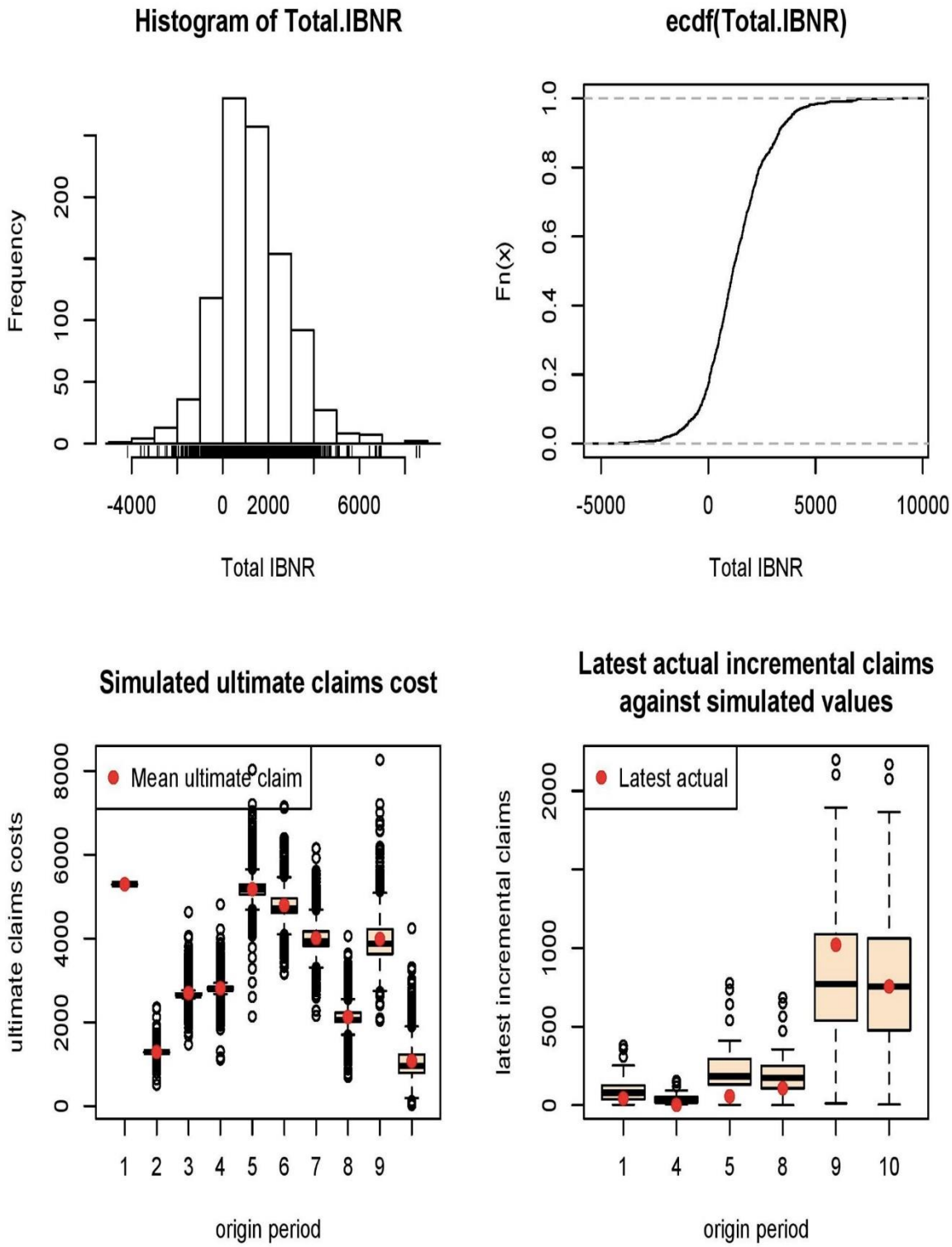



**Figure 6**

The above Figure 6 shows four graphs, starting with a histogram of the total simulated IBNRs over all origin periods, including a rug plot; a plot of the empirical cumulative distribution of the total IBNRs over all origin periods; a box-whisker plot of simulated ultimate claims costs against origin periods; and a box-whisker plot of simulated incremental claims cost for the



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latest available calendar period against actual incremental claims of the same period. In the last plot the simulated data should follow the same trend as the actual data, otherwise the original data might have some intrinsic trends which are not reflected in the model.

Quantiles of the bootstrap IBNR can be calculated via the quantile function:

**$By Origin**

| | IBNR 75% | IBNR 95% | IBNR 99% | IBNR 99.5% |
|---|---|---|---|---|
| 1 | 0.000000e+00 | 0.0000 | 0.0000 | 0.0000 |
| 2 | 1.052887e-03 | 86.19852 | 339.0520 | 384.4655 |
| 3 | 3.095093e-07 | 183.59047 | 628.3560 | 808.0620 |
| 4 | 2.030076e-28 | 105.92603 | 368.5968 | 486.2223 |
| 5 | -3.922391e-01 | 195.11278 | 486.0321 | 616.0637 |
| 6 | -2.167965e-01 | 39.83976 | 253.0985 | 312.9611 |
| 7 | 4.679461e+01 | 403.55723 | 780.7814 | 1,077.9382 |
| 8 | 1.495157e+02 | 623.96928 | 1,199.9575 | 1,322.9916 |
| 9 | 4.815752e+02 | 1,118.06372 | 1,880.8418 | 2,053.4537 |
| 10 | 2.321587e+03 | 3,621.97878 | 4,766.9408 | 4,988.0421 |

| **Totals** | |
|---|---|
| IBNR 75%: | 2,469.290 |
| IBNR 95%: | 4,219.778 |
| IBNR 99%: | 5,730.495 |
| IBNR 99.5%: | 6,075.568 |

The distribution of the IBNR appears to follow a log-normal distribution, so let's fit it:

| meanlog | sdlog |
|---|---|
| 7.27139285 | 0.93227773 |
| (0.03226280) | (0.02281325) |

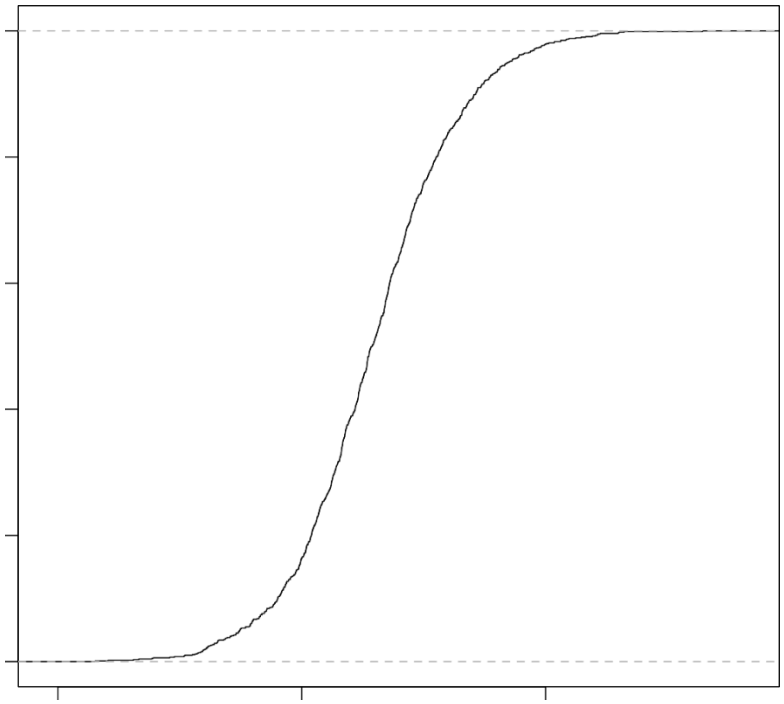


**Figure 7: ecdf(B$IBNR.Totals)**

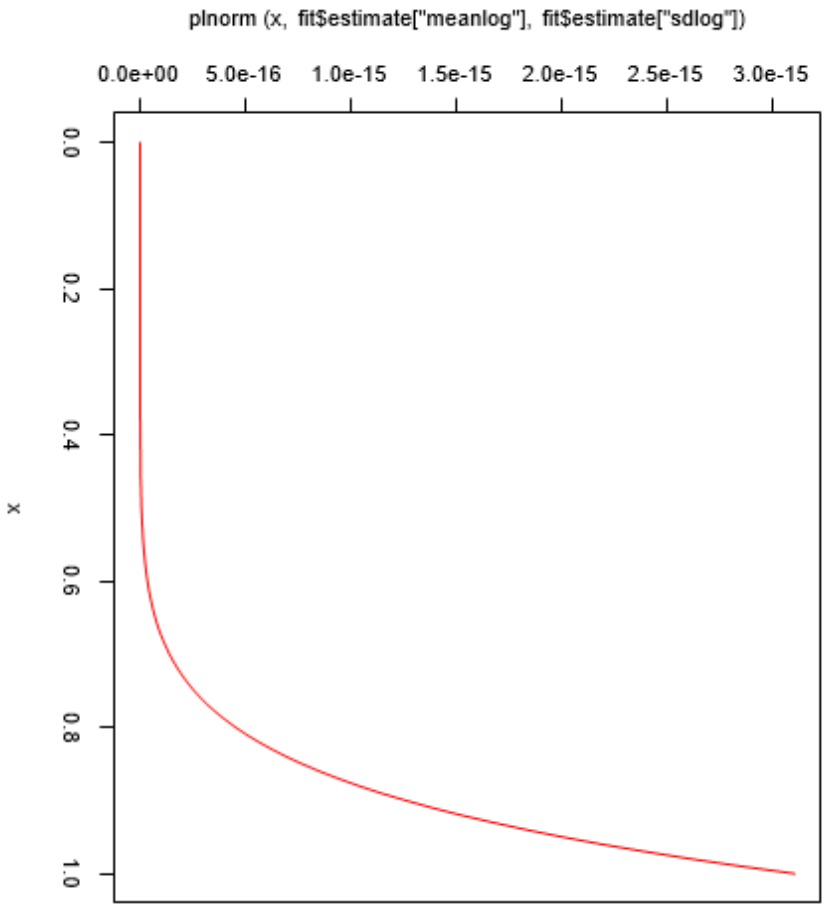



**Figure 8**



**URL:** http://dx.doi.org/10.14738/abr.116.14871

## CONCLUSIONS

1. The Loss Development Factor (LDF) is slightly above unity, i.e., 1.000105, which shows an increasingly positive trend for I.B.N.R.'s;
2. The claim amount for the last development period is estimated by both Mack and Bootstrap chain ladder methods at 34,412;
3. The predicted ultimate claim is estimated 35,987 under chain ladder method, Mack chain ladder estimated it at 35,984.28, while Bootstrap chain ladder method showed 35,891;
4. The predicted I.B.N.R. reserve was estimated at 1,572.28 under the Mack chain ladder method and 1,479 under Bootstrap chain ladder method;
5. Since the coefficient of variation of I.B.N.R.'s was estimated in absolute value well above 25%, i.e., 90%, we followed the Bootstrap chain ladder method, which also justified the increasingly positive trend of I.B.N.R.'s;
6. Hence, the results do not follow the trend for a hardening market in the 2022/23 renewal.

## FURTHER CONSIDERATION

Similar analysis of the open years for the pool claims has shown the following results:

- 2019/20: 1.0127889, i.e., a 1.28% increase
- 2020/21: 1.059312, i.e., a 5.93% increase

The respective results followed by the International Group of P&I Clubs were:

- 2019/20: renewal as before, i.e., no increase
- 2020/21: 1.4% increase

Ultimately, had the International Group of P. & I. Clubs adopted a slightly more aggressive renewal policy in the last two (2) years, this could have been sufficient to cover the increased reserves required following the recent substantial claims, thus allowing for a more rational strategy in the forthcoming 2022/23 renewal.

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